

## Homework 2: Gauss-Jordan elimination

This homework is due on Wednesday, February 1, respectively on Thursday February 2, 2017.

- 1 Row reduce the following matrices  $A, B$ . Try to do it in as few steps as possible.

$$\text{a) } A = \begin{bmatrix} 6 & 12 & 18 & 24 & 30 & 36 \\ 5 & 10 & 15 & 20 & 25 & 30 \\ 4 & 8 & 12 & 16 & 20 & 24 \\ 3 & 6 & 9 & 12 & 15 & 18 \\ 2 & 4 & 6 & 8 & 10 & 12 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}, \text{ b) } B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

- 2 Solve the system of equations  $A\vec{x} = \vec{b}$  for the Hadamard square

$$A = \begin{bmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{bmatrix}, \vec{b} = \begin{bmatrix} 10 \\ 14 \\ 18 \\ 22 \end{bmatrix}$$

by row reducing the augmented  $4 \times 5$  matrix  $B = [A|\vec{b}]$ . The Hadamard

product of two matrices is  $(U * V)_{ij} = U_{ij}V_{ij}$ . In this case we took  $B_{ij} = i + j - 1$  and formed  $A = B * B$ . This is of course not the same than

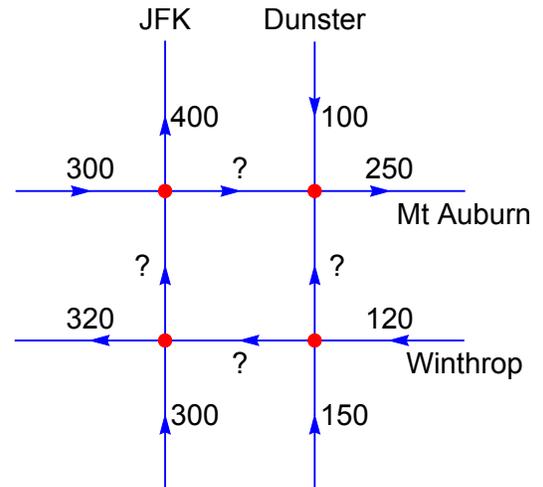
$B^2 = B.B$ .

- 3 Find all solutions of the following system of equations using Gauss-Jordan elimination. As usual, you need to show all work.

$$\begin{aligned} 4x_1 + 3x_2 + 2x_3 - x_4 &= 4 \\ 5x_1 + 4x_2 + 3x_3 - x_4 &= 4 \\ -2x_1 - 2x_2 - x_3 + 2x_4 &= -3 \\ 11x_1 + 6x_2 + 4x_3 + x_4 &= 11 \end{aligned}$$

4 Two  $n \times m$  matrices in reduced row-echelon form are called **of the same type** if they contain the same number of leading 1's in the same positions. For example,  $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  are of the same type. How many types of  $2 \times 2$  matrices in reduced row-echelon form are there?

5 The traffic on the streets near Harvard is indicated on the figure. Assuming that the total traffic leaving a node is the amount entering it, what can you say about the traffic at the four locations indicated by question marks? What is the highest and the lowest possible traffic volume at each location?



## Main definitions

The **Gauss-Jordan Elimination** process brings a matrix into **reduced row echelon form**. It consists of **elementary row operations**: **Swap** two rows. **Scale** a row. **Subtract** a multiple of a row from an other.

The **row-reduced matrix**  $\text{rref}(A)$  has three properties:

- 1) if a row has nonzero entries, then the first nonzero entry is **1** (“**leading 1**”).
- 2) if a column contains a leading 1, then the other entries in that column are 0.
- 3) if a row has a leading 1, then every row above has a leading 1 to the left.

The number of leading 1 in  $\text{rref}(A)$  is called the **rank** of  $A$ .