## Homework 1: Linear Equations

This homework is due on Monday, January 30, respectively Tuesday January 31, 2017. Homework is due at the beginning of each class in the classroom.

1 Find all solutions of the linear system $\begin{vmatrix} 2x - y + 2z & = -3 \\ x + y - z & = 6 \\ x + 2y - 4z & = 16 \end{vmatrix}$

2 A secret "fraction lab" near Boston has discovered three nonzero fractions x, y, z such that their sum adds up to 1. One of the fractions is twice as big the other. You also know that one of the fractions is three times as big as an other. The lab has found two solutions but knows that there are four. Can you find them all?
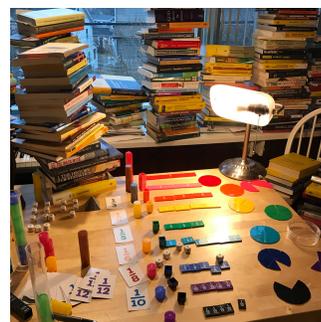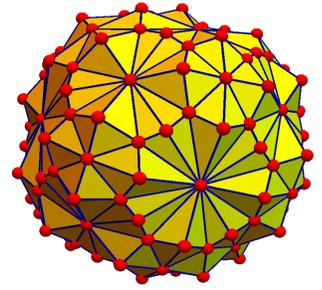
3 A 10 km trip from the Swiss waterfall "Rheinfall" to the village "Rheinau" takes 30 minutes. The return trip takes an hour. How fast is the speed v (in km/h) of the boat traveling relative to the water, and how fast is the speed s (in km/h) of the river?

4 On a heating mesh, the temperature at exterior mesh points is $0, 400$ or 800 F as given in the picture. In thermal equilibrium, each interior mesh point has the average of the temperatures at the 4 adjacent points. For example $T_2 = (T_3 + T_1 + 400 + 0)/4$. Find the temperatures T_1, T_2, T_3.
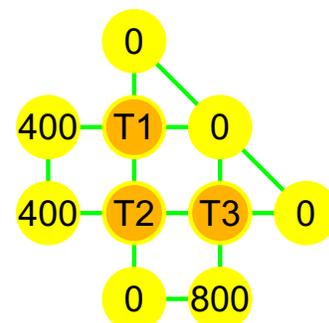
5 A polyhedron has v vertices, e edges and f triangular faces. Euler proved his famous formula $v - e + f = 2$. There is an other relation, $3f = 2e$ called a Dehn-Sommerville relation which always holds. The number f is called the area. You get a polyhedron with area 288. Write down a system of equations in matrix form $Ax = b$. Then determine the number of vertices and edges.