

Homework 29: Fourier I

This homework is due on Wednesday, April 20, respectively on Thursday, April 21, 2016.

- 1 a) Find the angle between the functions $f(x) = x$ and $g(x) = x^4$.
 b) Project $f(x) = \cos^3(x)$ onto the plane spanned by $\sin(x)$, $\cos(x)$.
 c) Find the length of the function $f(x) = e^x$ in C_{per}^∞ .
- 2 Verify that the functions $\cos(nx)$, $\sin(nx)$, $1/\sqrt{2}$ form an orthonormal family.
- 3 Find the Fourier series of the function $f(x) = 7 - |3x|$.
- 4 Find the Fourier series of the function $4 \cos^2(x) + 5 \sin^2(11x) + 10$.
- 5 Find the Fourier series of the function $f(x) = 9|\sin(x)|$.

Fourier Series I

We write 2π periodic functions always as functions on $[-\pi, \pi]$. In C_{per}^∞ we have the inner product $\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x) dx$. This defines the **length** $|f| = \sqrt{\langle f, f \rangle}$ and angle $\cos(\alpha) = \langle f, g \rangle / (|f||g|)$. The functions $\sin(nx)$, $\cos(nx)$, $1/\sqrt{2}$ form an orthonormal basis in C_{per}^∞ in the sense that they are linearly independent and span the space. Any function f can be written uniquely as a linear combination of these terms. With the Fourier coefficients $a_0 = \langle f, 1/\sqrt{2} \rangle$, $a_n = \langle f, \cos(nx) \rangle$ and $b_n = \langle f, \sin(nx) \rangle$, we can write

$$f = a_0/2 + \sum_n a_n \cos(nx) + \sum_n b_n \sin(nx)$$

This is called the Fourier series of f .