Homework 17: Determinants II

This homework is due on Monday, March 21, respectively on Tuesday, March 22, 2016.

1 a) Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 4 & 4 & 4 & 5 \\ 6 & 6 & 8 & 9 & 10 \\ 11 & 11 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 0 \end{bmatrix}.$$

b) Find the determinant of A^{-4}.

2 a) Find the determinant of

$$B = \begin{bmatrix} 3 & 2 & 0 & 0 & 0 & 0 \\ 3 & 3 & 2 & 0 & 0 & 0 \\ 3 & 3 & 3 & 2 & 0 & 0 \\ 3 & 3 & 3 & 3 & 2 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 \\ 3 & 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the determinant of $3B$.

3 a) Find the determinant of

$$A = \begin{bmatrix} 3 & 1 & 1 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 & 2 \\ 1 & 0 & 3 & 2 & 2 & 2 \\ 0 & 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

b) Find the determinant of A^{10}.

4 Argue geometrically why the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

has maximal absolute determinant $|\det(A)|$ among all matrices with entries in $\{-1, 1\}$.

5 a) Find A, B such that $\det(A + B) \neq \det(A) + \det(B)$.
b) What possible determinant values does an orthogonal matrix have?
c) Verify that $|\det(A)|$ only depends on R if $A = QR$.

Determinants II

Determinants can be computed using row reduction: If during row reduction m swapping operations have occured and the scaling factors are $c_1, ..., c_k$, then

$$\det(A) = \frac{(-1)^m}{c_1 \cdots c_k} \det(\text{rref}(A))$$

Here are some more properties:

- $|\det(A)|$ is the volume of a parallel eppiped
- $\det(AB) = \det(A)\det(B)$
- $\det(A^T) = \det(A)$
- $\det(A^n) = (\det(A))^n$
- $\det(A^{-1}) = 1/\det(A)$