

Homework 15: Data fitting

This homework is due on Wednesday, March 9, respectively on Thursday, March 10, 2016.

- 1 a) Find the least square solution x^* of the system $Ax = b$ with

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}, \text{ and } b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

- b) What is the matrix P which projects on the image of A ?

- 2 Find the function $y = f(x) = ax + bx^3$, which best fits the data

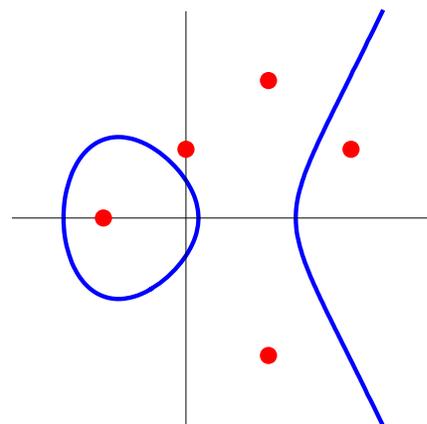
x	y
-1	1
1	3
-2	10

- 3 A curve of the form

$$y^2 = x^3 + ax + b$$

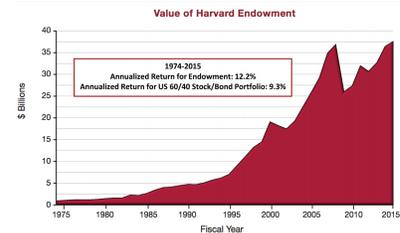
is called an **elliptic curve** in Weierstrass form. Elliptic curves are important in cryptography. Use data fitting to find the best parameters (a, b) for an elliptic curve given the following points:

$$\begin{aligned} (x_1, y_1) &= (1, 2) \\ (x_2, y_2) &= (-1, 0) \\ (x_3, y_3) &= (2, 1) \\ (x_4, y_4) &= (0, 1) \end{aligned}$$



4 A graphic from the Harvard Management Company Endowment Report of September 2015 is shown to the right. Assume we want to fit the growth using functions $1, x, x^2$ and assume the years are numbered starting with $1990 = 0, 1995 = 1, 2000 = 2, 2005 = 3, 2010 = 4, 2015 = 5$. What is the best parabola $a + bx + cx^2 = y$ which fits these data?

quintennium	billions
0	5
1	15
2	19
3	23
4	27
5	37



5 Find the circle $a(x^2 + y^2) + b(x + y) = 1$ which best fits the data

x	y
0	1
-1	0
1	-1
1	1

In other words, find the least square solution for the system of equations for the unknowns a, b which aims to have all 4 data points (x_i, y_i) on the circle.

Data fitting

Given a system $Ax = b$. Any solution of $(A^T A)x = A^T b$ is called a **least square solution** (these always exist). (Reason: solve $A^T(Ax - b) = 0$ for x , assuring that $Ax - b$ is perpendicular to $\text{im}(A)$.) The least square solution is unique if A has a trivial kernel. In that case $x = (A^T A)^{-1} A^T b$. The matrix $A(A^T A)^{-1} A^T$ is now the projection matrix onto $\text{im}(A)$. If the columns of A are orthonormal, this simplifies to $P = AA^T$.