Homework 11: Linear spaces

This homework is due on Friday, February 26, respectively on Tuesday, March 1, 2016.

1 Which of the following spaces are linear spaces?
   a) The set of all 3×3 matrices.
   b) The set of all polynomials of degree exactly 2.
   c) The set of all invertible 2×2 matrices.
   d) The set of all diagonal 3×3 matrices.
   e) The set of all 2×2 matrices for which the sum of the diagonals is zero.
   f) The set of all functions f in $C(R)$ for which $f(5) = 0$.
   g) The set of all functions f in $C^\infty(R)$ in for which $f'(0) = 1$.
   h) The set of all functions f in $C^\infty(R)$ for which $f''(2) = 0$.

2 a) Find a basis for all the 2×2 rotation dilation matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$.

   b) Find a basis for all the 2×2 reflection dilation matrices $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

   c) Find a basis for all the 2×2 matrices for which Ae_1 is parallel to e_1.
   d) Find a basis for all the diagonal 3×3 matrices.
   e) Find a basis for all the 2×2 dilation matrices.

3 Find a basis for all the 2×2 matrices A for which

$$A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4 A function is called **even** if $f(-x) = f(x)$ for all x.
   A function is called **odd** if $f(-x) = -f(x)$ for all x.

a) Find a basis for all the even polynomials in $P_6(R)$

b) Find a basis for all the odd polynomials in $P_5(R)$.

c) Find a basis for all polynomials f in $P_4(R)$ for which $f(0) = 0$.

5 a) Find a concrete matrix A with no zeros for which $A\vec{v} = 0$,

where $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

b) Let V be the set of 3×3 matrices for which $\vec{v}$ is in the kernel. Is this a linear space?

Linear space

A set X of objects which can be added and scaled and which contain a zero element is called a **linear space**. To check verify (i) 0 is in X, (ii) if x and y are both in X, then $x + y$ is in X, (iii) if x is in X, then λx is in X for all constants c. There are three important classes of linear spaces: first the subspaces of R^n as treated before, the second class is the set $M(n, m)$ of all $n \times m$ matrices. Finally, there is the class $C(R)$ for all continuous functions on the real line. It contains the linear space $C^\infty(R)$ for all smooth functions, functions which can be differentiated arbitrary often. We also write $P_n(R)$ for the set of polynomials of degree less or equal to n. It has dimension $n + 1$. In order to check whether a subset of functions or matrices or vectors in R^n are a linear space, check the three things (i),(ii),(iii). Note that in the case of functions, 0 is the function which is 0 for all x.