Homework 10: Coordinates

This homework is due on Wednesday, February 24, respectively on Thursday, February 25, 2016.

1 What are the $\mathcal{B}$-coordinates of the vector $\vec{v}$ in the basis $\mathcal{B}$.

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \mathcal{B} = \{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \} ?$$

2 What is the matrix B for the transformation $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ in the

basis $\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \}$.

3 Chose a suitable basis to solve the following two problems:
a) Find the matrix A which belongs to a reflection at the plane $x + y + 2z = 0$.
b) Find the matrix A which belongs to the reflection at the line

spanned by $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

4 Find the matrix A corresponding to the orthogonal projection

onto the plane spanned by the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

5 The whole plane is covered with regular hexagons "Graphene",

where the first basis vector is $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. a) Find w so that $\mathcal{B} = $

$\{v, w\}$ is the basis as seen in the picture.

b) What are the standard coordinates of the vector given in the $\mathcal{B}$ basis as $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$?

c) Is the point with $\mathcal{B}$ coordinate $\begin{bmatrix} 17 \\ 21 \end{bmatrix}$ a vertex of a hexagon or the center of one?
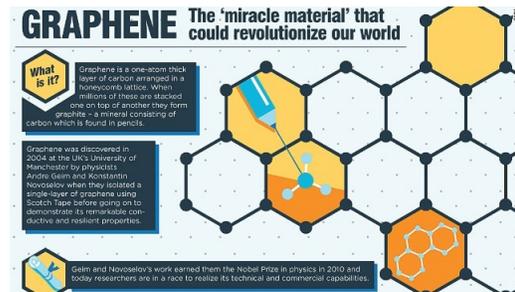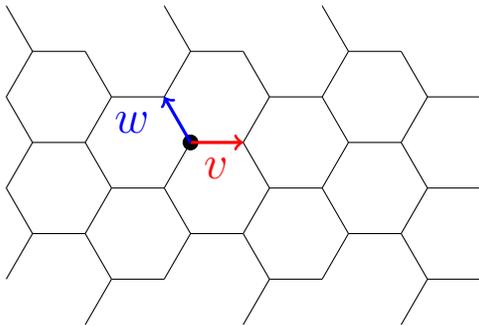
Source: CNN

Coordinates

Given a basis $\mathcal{B} = \{\vec{v}_1, \ldots, \vec{v}_n\}$ of a linear space V, every $\vec{w}$ in V can be written as $\vec{w} = c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$, where c_i are the **coordinates** of v. The basis defines a matrix $S =$

$$\left\{ \begin{bmatrix} | \\ \vec{v}_1 \\ | \end{bmatrix}, \begin{bmatrix} | \\ \vec{v}_2 \\ | \end{bmatrix} \cdots \begin{bmatrix} | \\ \vec{v}_n \\ | \end{bmatrix} \right\}.$$ Since $S\vec{c} = \vec{w}$ we get $\boxed{\vec{c} = S^{-1}\vec{w}}$.

If A is a matrix given in the standard basis $e_1, \ldots, e_n$ and B is the matrix written in the basis $\mathcal{B}$, then $\boxed{B = S^{-1}AS}$.
We say B is **similar** to A. Why do we want to change basis? Because it is convenient: for example if v_1, v_2 are non-parallel vectors in a plane and v_3 is perpendicular to the plane then a projection onto the plane is the matrix $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. The matrix in the standard basis is then $A = SBS^{-1}$.