

Homework 7: Image and Kernel

This homework is due on Wednesday, February 17, respectively on Tuesday February 16, 2016.

- 1 Find the kernel of the transformation $x \rightarrow Ax$, then write down a set of vectors which span the image of A .

a) $\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 3 & 4 & 3 & 2 \\ 3 & 4 & 5 & 4 & 3 \\ 4 & 5 & 6 & 5 & 4 \end{bmatrix}$, b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, c) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$.

d) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

- 2 a) Give an example of a transformation from R^6 to R^4 for which

the image is a plane spanned by the two vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

- b) Express the kernel of the 1×3 matrix $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ as the image of a 3×2 matrix B .

- 3 a) What is the image and kernel of the rotation–dilation $\begin{bmatrix} 5 & 12 \\ -12 & 5 \end{bmatrix}$?

- b) What is the image and kernel of the projection $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$?

- c) What is the image and kernel of the reflection $\frac{1}{13} \begin{bmatrix} 5 & 12 \\ 12 & -5 \end{bmatrix}$?

- d) What is the image and kernel of the shear $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$?

e) What is the image and kernel of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

4 Let A be an arbitrary matrix and $B = \text{rref}(A)$.

a) Is it true that $\text{im}(A) = \text{im}(B)$? Explain why or why not.

b) Is it true that $\ker(A) = \ker(B)$? Explain why or why not.

c) Be creative and find a 3×3 or 4×4 matrix for which $\text{im}(A) = \ker(A)$.

5 Let A be a $n \times n$ matrix.

a) What is the relation between the kernel of A and the kernel of A^3 ?

b) What is the relation between the image of A and the image of A^3 ?

c) What is the relation between the kernel of A and the kernel of $A^3 + A$?

d) What is the relation between the image of A and the image of $A^3 + A$?

Image and kernel

The **image** of a linear map $x \rightarrow Ax$ is the set of all vectors Ax . The **kernel** is the set of vectors x which satisfy $Ax = 0$. The columns of A **span** the image of A . Every $x \in \text{im}(A)$ can be written as a linear combination of column vectors. The image and kernel are both linear spaces: they are closed under addition, scalar multiplication and contain the zero vector. The kernel of a $n \times n$ matrix is $\{0\}$ if and only if A is invertible if and only if the image is R^n if and only if $\text{rref}(A) = 1$.