Homework 6: Matrix Algebra

This homework is due on Friday, February 12, respectively on Tuesday February 16, 2016.

1 For each pair of matrices A and B, compute both AB and BA.

a) $A = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & \frac{1}{2} \end{bmatrix}$.

b) $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 6 & 11 \end{bmatrix}$.

2 Find a 2×2 matrix A with no 0 entries such that $A^2 = 0$.

3 a) Find the inverse of the matrix A made from the first 5 rows of

Pascal's triangle. $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$.

b) The following $0 - 1$ matrix B has the property that the inverse

is again a $0 - 1$ matrix: $B = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$. Find the inverse.

4 a) Assume $A^7 = A \cdot A \cdot A \cdot A \cdot A \cdot A \cdot A$ is the identity matrix. Can you find a simple formula in terms of A which gives A^{-1}?

b) Find a transformation in the plane which has the property that $A^7 = 1$. Find A^{-1}.

5 a) Assume A is small enough so that $B = 1 + A + A^2 + A^3 + \dots$ converges. Verify that B is the inverse of $1 - A$.

b) Use Mathematica to plot A and B^{-1} for the 100 x 100 matrices defined by $A_{nm} = \sin(nm + n^2 + m^2)$ and $B_{nm} = \sin(nm)$.

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(* Example to plot a matrix A(n,m)=n+m *)
MatrixPlot[Inverse[Table[N[n+m],{n,100},{m,100}]]]
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The reason for the N[...] part is to get a real matrix and not integer matrix. Mathematica might not succeed with getting an integer answer in the HW case.

Matrix Algebra

Matrices can be added, multiplied with a scalar. One can also form the product of two matrices $A \cdot B$ as well as the inverse matrix A^{-1} if the matrix is invertible. These operations constitute the **matrix algebra**. It behaves like the algebra of real numbers but the multiplication is no more commutative in general. Besides the matrix 0 where all entries are zero there are other matrices which are not invertible. We write 1 for the identity matrix which has 1 in the diagonal and 0 everywhere else. Now $A1 = A$.

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A={{5,2},{3,4}}; MatrixInverse[A]
MatrixPower[A,7]
IdentityMatrix[2] + A
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