Homework 5: transformations in geometry

This homework is due on Wednesday, February 10, respectively on Thursday February 11, 2016.

1 a) Find the 2×2 rotation dilation matrix which rotates by $45°$ **clockwise** and scales by a factor $3\sqrt{2}$.
b) Find the rotation dilation matrix which rotates around the origin by $60°$ counterclockwise and scales by a factor 2. c) Find the reflection matrix at the line $y = -x$ in the plane.
d) Find the projection matrix onto the line $y = -x$ in the plane.

2 Name the following transformations and give a reason. Choose from Dilation, Shear, Rotation, Projection, Reflection, and Rotation with Dilation. (Note: not all transformations might occur.)
a) $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, b) $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, c) $A = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$, d) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, e) $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, f) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

3 Pair the linear transformations given by the matrices A, B, C, D, E below with one of the following types:
a) a rotation around a line
b) an orthogonal projection onto a line
c) reflection about a line
d) a reflection about a plane
e) an orthogonal projection onto a plane

$$A = \frac{1}{9}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix}, B = \frac{1}{9}\begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{bmatrix}, C = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & -2 \end{bmatrix},$$

$$D = \frac{1}{3}\begin{bmatrix} 2 & -2 & -1 \\ -2 & -1 & -2 \\ -1 & -2 & 2 \end{bmatrix}, E = \frac{1}{5}\begin{bmatrix} 3 & 0 & 4 \\ 0 & 5 & 0 \\ -4 & 0 & 3 \end{bmatrix}.$$

4 Assume $a^2+b^2 = 1$. One of the three transformations is a rotation, the other is a reflection about a line, the third is an orthogonal projection onto a line. Which is which? Find the inverse in the case of rotation and reflection.
a) $C = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$. b) $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. c) $B = \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$. d) For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is defined as $\det A = ad - bc$. Find the determinant of a shear, of a rotation, of a reflection about a line, of reflection in the origin, a projection onto the x axis, a rotation-dilation matrix with parameters (a, b).

5 The matrix multiplication introduced in the next lecture gives the entry ij of AB as the dot product of the i'th row of A with j'th column of B. Verify that the product of a reflection dilation $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ with an other reflection dilation $\begin{bmatrix} c & d \\ d & -c \end{bmatrix}$ is a rotation dilation. What is the rotation angle and what is the scaling factor? Optional: try to understand this geometrically.

Transformations to know

Make your own personal collection of transformations like horizontal and vertical shear, dilations=scalings, rotation in the plane and space, projection onto a plane, line, reflection about plane, a line or point or combinations.