Homework 1: Linear Equations

This homework is due on Monday, February 1, respectively Tuesday February 2, 2016. Homework is due at the beginning of each class in the classroom.

1 Find all solutions of the linear system $\begin{vmatrix} 2x + 3y + 3z &= 39 \\ x + 2y + 3z &= 32 \\ x + y + 4z &= 35 \end{vmatrix}$

2 Jake and Maggie are siblings. Maggie has twice as many brothers as sisters and Jake has just as many brothers as sisters. How many kids are there in this family? (Any resemblance to the Gyllenhaal siblings is purely coincidental).
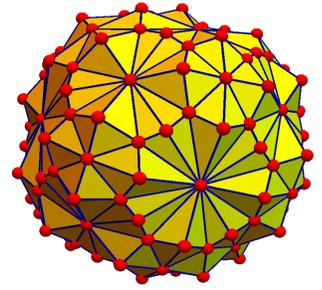
3 A 10 km trip from the Swiss waterfall "Rheinfall" to the village "Rheinau" takes 30 minutes. The return trip takes an hour. How fast is the speed v (in km/h) of the boat traveling relative to the water, and how fast is the speed s (in km/h) of the river?

4 On a heating mesh, the temperature at exterior mesh points is $0, 200$ or 400 F as given in the picture. In thermal equilibrium, each interior mesh point has the average of the temperatures at the 4 adjacent points. For example $T_2 = (T_3 + T_1 + 200 + 0)/4$. Find the temperatures T_1, T_2, T_3.
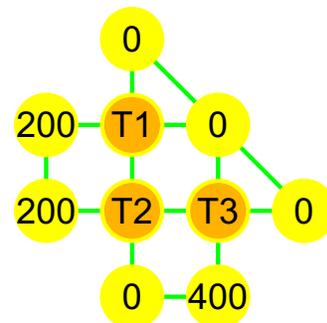
5 A polyhedron has v vertices, e edges and f triangular faces. Euler proved his famous formula $v - e + f = 2$. There is an other relation, $3f = 2e$ called a Dehn-Sommerville relation which always holds. The number f is called the area. You get a polyhedron with area 288. Write down a system of equations in matrix form $Ax = b$. Then determine the number of vertices and edges.

A **linear equation** for finitely many variables $x_1, x_2, \ldots, x_n$ is an equation of the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b .$$

If m equations of this type are given, we have a **system of linear equations**. It can be written in matrix form $A\vec{x} = \vec{b}$, where $\vec{x}$ is a column vector containing the n variables and the $m \times n$ matrix A lists all the $m \cdot n$ coefficients. Finally, $\vec{b}$ is the column vector listing the values to the right. For example, the system $x + 2y + z = 8, 3x - y - 7z = 4$ of equations can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix} .$$

Unlike in the homework examples given here, the number of variables and equations do not always have to be the same.