

Homework 30: Fourier II

This homework is due on Friday, April 24, respectively on Tuesday, April 23, 2015.

- 1 Find the Fourier series of the function which is 1 on $[0, \pi/2]$ and zero everywhere else.
- 2 Use Parseval to find $\int_{-\pi}^{\pi} f(x)^2 dx$ for $f(x) = \cos(11x) + \cos(13x) + 2\sin(17x) - \cos(19x) + 4\cos(1111x)$.
- 3 Compute both sides of the Parseval identity for $f(x) = x + |x|$.
- 4 Find $\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = 1/4 + 1/16 + 1/36 + \dots$ from the known formula of $\sum_n \frac{1}{n^2}$ and use this to compute $\sum_n \frac{1}{(2n+1)^2}$.
- 5 This HW is a preparation for PDEs and consists of reminders. All statements are pretty straight forward if we work with functions on $[-\pi, \pi]$ described by Fourier series:
 - a) Verify that the Fourier basis $\mathcal{B} = \{1, \cos(nx), \sin(nx)\}$ consists of eigenfunctions of D^2 .
 - b) What are the corresponding eigenvalues?
 - c) Show that every eigenfunction of D^2 is either constant or of the form $a \cos(nx) + b \sin(nx)$ for some n .
 - d) What are the eigenvalues of $D^2 + D^4 + 6$ on the subspace of C_{per}^{∞} consisting of odd functions?

Fourier Series II

Recall that the Fourier coefficients of a function $f \in C_{per}^\infty$ are defined as $a_0 = \langle f, 1/\sqrt{2} \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)/\sqrt{2} dx$, $a_n = \langle f, \cos(nt) \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$, $b_n = \langle f, \sin(nt) \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$. These are just inner products $a_n = \langle f, \cos(nx) \rangle$ and $b_n = \langle f, \sin(nx) \rangle$ and

$$f = a_0/\sqrt{2} + \sum_n a_n \cos(nx) + \sum_n b_n \sin(nx)$$

is the Fourier series of f . The Parseval identity is $\|f\|^2 = a_0^2 + \sum_{k=1}^{\infty} a_k^2 + b_k^2$. It is an extension of the Pythagoras theorem.