

Homework 23: Symmetric matrices

This homework is due on Monday, April 6, respectively on Tuesday, April 7, 2015.

- 1 Give a reason why its true or provide a counterexample.
 - a) The sum of two symmetric matrices is symmetric.
 - b) The product of two symmetric matrices is symmetric.
 - c) The inverse of an invertible symmetric matrix is symmetric.
 - d) If B is an arbitrary $n \times m$ matrix, then $A = B^T B$ is symmetric.
 - e) If A is similar to B and A is symmetric, then B is symmetric.
 - f) $A = S^{-1} B S$ with $S^T S = I_n$, A symmetric $\Rightarrow B$ is symmetric.
 - g) Only the zero matrix is both anti-symmetric and symmetric.
- 2 Find all the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2016 & 2 & 3 & 4 & 5 \\ 2 & 2019 & 6 & 8 & 10 \\ 3 & 6 & 2024 & 12 & 15 \\ 4 & 8 & 12 & 2031 & 20 \\ 5 & 10 & 15 & 20 & 2040 \end{bmatrix}.$$

- 3 a) Find the eigenvalues and an orthonormal eigenbasis to $A =$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

- b) Use technology to find the eigenvalues and eigenvectors of the

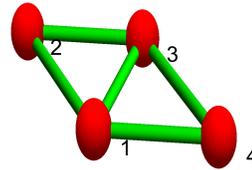
$$\text{GCD matrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 2 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 & 5 \end{bmatrix}.$$

4 Group matrices which are similar.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

5 Find the eigenvalues and eigenvectors of the Laplacian of the Kite graph. The Laplacian of a graph with n nodes is the $n \times n$ matrix A which for $i \neq j$ has $A_{ij} = -1$ if i, j are connected and 0 if not. The diagonal entries A_{ii} are chosen so that each row adds up to 0.

$$A = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$



Symmetric matrices

A matrix A is **symmetric** if $A^T = A$ and anti-symmetric if $A^T = -A$. Projections or reflections are symmetric. Symmetric matrices appear in physics or statistics: observables like energy, position or momentum matrices are symmetric and correlation matrices. The spectral theorem tells that a symmetric matrix has real eigenvalues, that it has an orthonormal eigenbasis and can be diagonalized as $S^{-1}AS = B$ with an orthogonal matrix S .