

Homework 22: Stability

This homework is due on Friday, April 3, respectively on Tuesday, April 7, 2015.

1 Determine the stability of the dynamical system $x(t+1) = Ax(t)$:

a)
$$\begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.2 & 0.3 & 0.4 \\ 0.2 & 0.3 & 0.4 \end{bmatrix}.$$

b)
$$\begin{bmatrix} 0.9 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0.2 \end{bmatrix}.$$

2 For which constants a is the system $x(t+1) = Ax(t)$ stable?

a)
$$A = \begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}.$$

b)
$$A = \begin{bmatrix} a & a \\ a & a \end{bmatrix}.$$

c)
$$A = \begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix}.$$

3 Explain for which k the drawing rule

$$x(t+1) = x(t) - ky(t)$$

$$y(t+1) = y(t) + kx(t+1)$$

produces trajectories which are ellipses.

4 Find the eigenvalues of

$$\begin{bmatrix} 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & a \\ a & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For which a is the corresponding system stable?

5 In the following, answer the question and give a short explanation.

We say A is stable if the origin $\vec{0}$ is a stable equilibrium.

- a) True or false: A is stable if and only if A^T is stable.
- b) True or false: A is stable if and only if A^{-1} is stable.
- c) True or false: A is stable if and only if $A + 1$ is stable.
- d) True or false: A is stable if and only if A^2 is stable.
- e) True or false: A is stable if $A^2 = 0$.
- f) True or false: A is unstable if $A^2 = A$.
- g) True or false: A is stable if A is diagonalizable.
- h) True or false: any orthogonal matrix is stable.
- i) True or false: every shear is stable.
- j) True or false: the zero matrix is stable.

Stability

A discrete dynamical system $x(t+1) = Ax(t)$ is **asymptotically stable** if $x(t) \rightarrow 0$ as $t \rightarrow \infty$ for all initial conditions $x(0)$. A system is asymptotically stable if and only all eigenvalues of A have absolute value $|\lambda_j| < 1$. For example, a rotation dilation A with first column $Ae_1 = [a, b]^T$ is stable if and only if $a^2 + b^2 < 1$. We often just say “ A is stable” rather than “the origin is stable for the discrete dynamical system $x \mapsto Ax$ ”.