

Homework 19: Eigenspaces

This homework is due on Friday, March 27, respectively on Tuesday, March 31, 2015.

- 1 Find all eigenvalues and eigenvectors for the following matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

What are the algebraic and geometric multiplicities?

- 2 Assume you know that a 2 times 2 matrix A has an eigenspace spanned by $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and an eigenspace spanned by $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and that the eigenvalues are 7 and 11. Find the matrix A .
- 3 a) Verify that A and A^T have the same eigenvalues.
b) Assume A is invertible. What is the relation between the eigenvalues of A and A^{-1} ?



- 4 The vector $A^n b$ gives pollution levels in the Silvaplana, Sils and St Moritz lake n weeks after an oil spill. The matrix is $A = \begin{bmatrix} 0.7 & 0 & 0 \\ 0.1 & 0.6 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix}$ and $b = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$ is the initial pollution level. Find a closed form solution for the pollution after n weeks.

5 a) Find the eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & -2 \\ 3 & 6 & -5 \end{bmatrix} .$$

- b) Find the eigenvectors of A^{10} , where A is the previous matrix.
c) Find the eigenvectors of $(A^T)^{-1}$, where A is the previous matrix.

Eigenspaces

A nonzero vector \vec{v} is called an **eigenvector**, if $Av = \lambda v$ for some λ . The set of eigenvectors is called the **eigenspace** E_λ . It is the kernel of $A - \lambda I_n$. The dimension of the eigenspace is called the **geometric multiplicity** of λ . There is a general result which tells that the geometric multiplicity of λ is always smaller or equal to the algebraic multiplicity.

Recall that A is similar to B if there exists an invertible S such that $B = S^{-1}AS$. If A and B are similar, then they have the same characteristic polynomial, the same eigenvalues and algebraic multiplicities as well as the same geometric multiplicities. Similar matrices also have the same trace $\text{tr}(A) = \lambda_1 + \dots + \lambda_n$ as well as determinant $\det(A) = \lambda_1 \cdots \lambda_n$. These formulas hold in general if we allow the eigenvalues λ_i to be complex. More on complex eigenvalues next week.