

## Homework 17: Determinants II

This homework is due on Monday, March 23, respectively on Tuesday, March 24, 2015.

- 1 a) Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 4 & 5 \\ 6 & 6 & 8 & 9 & 10 \\ 11 & 11 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 0 \end{bmatrix}$$

- b) Find the determinant of  $A^{-2}$ .

- 2 a) Find the determinant of

$$B = \begin{bmatrix} 3 & 2 & 0 & 0 & 0 & 0 \\ 3 & 3 & 2 & 0 & 0 & 0 \\ 3 & 3 & 3 & 2 & 0 & 0 \\ 3 & 3 & 3 & 3 & 2 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 \\ 3 & 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

- b) Find the determinant of  $3B$ .

- 3 a) Find the determinant of

$$A = \begin{bmatrix} 3 & 1 & 1 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 & 2 \\ 1 & 0 & 3 & 2 & 2 & 2 \\ 0 & 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

- b) Find the determinant of  $A^{10}$ .

- 4 a) Find an example showing that  $\det(A + B) \neq \det(A) + \det(B)$ .  
b) How do you modify  $\det(\lambda A) = \lambda \det(A)$  to make it correct if  $A$  is a  $n \times n$  matrix?
- 5 a) Verify that if  $A + I_n$  is invertible and  $A - I_n$  is invertible, then  $A^2 - I_n$  is invertible.  
b) How are the determinants of two similar matrices related?

## Determinants II

Determinants can be computed using row reduction: If during row reduction of a matrix  $A$  we swap rows  $m$  times and scale rows  $k$  times, multiplying by the numbers  $c_1, \dots, c_k$ , then

$$\det(A) = \frac{(-1)^m}{c_1 \cdots c_k} \det(\text{rref}(A))$$

Here are some more properties:

- $\det(AB) = \det(A)\det(B)$
- $\det(A^T) = \det(A)$
- $\det(A^n) = (\det(A))^n$
- $\det(A^{-1}) = 1/\det(A)$