Homework 15: Data fitting

This homework is due on Wednesday, March 11, respectively on Thursday, March 12, 2015.

1 a) Find the least square solution x^* of the system $Ax = b$ with
$$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}, \text{ and } b = e_2.$$
b) What is the matrix P which projects on the image of A?

x	y
-1	1
1	3
0	10

2 Find the function $y = f(x) = ax^2 + bx^3$, which best fits the data

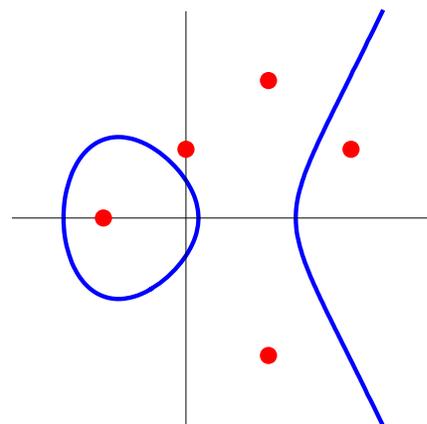**3** A curve of the form

$$y^2 = x^3 + ax + b$$

is called an **elliptic curve** in Weierstrass form. Elliptic curves are important in cryptography. Use data fitting to find the best parameters (a, b) for an elliptic curve given the following points:

$$(x_1, y_1) = (1, 2)$$
$$(x_2, y_2) = (-1, 0)$$
$$(x_3, y_3) = (2, 1)$$
$$(x_4, y_4) = (0, 1)$$

4 A graphic from the Harvard Management Company Endowment Report of September 2014 is shown to the right. Assume we want to fit the growth using functions $1, x, x^2$ and assume the years are numbered starting with $1990 = 0, 1995 = 1, 2000 = 2, 2005 = 3, 2010 = 4, 2015 = 5$. What is the best parabola $a + bx + cx^2 = y$ which fits these data?

quintenium	billions
0	5
1	15
2	19
3	23
4	27
5	37

Total Value of the Endowment
(after distributions)
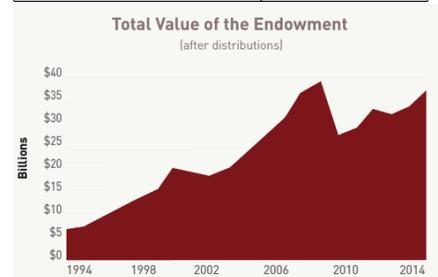
5 a) Draw the situation and find different regression lines which are optimal.
b) Now write down the corresponding fitting problem for linear functions $f(x) = ax + c = y$ by finding the matrix A and the vector b. What is going on?

For this fitting problem, the solution is not unique.

x	y
0	1
0	2
0	3

Data fitting

Given a system $Ax = b$. Any solution of $(A^T A)x = A^T b$ is called a **least square solution** (these always exist). (Reason: solve $A^T(Ax - b) = 0$ for x, assuring that $Ax - b$ is perpendicular to $\text{im}(A)$.) The least square solution is unique if A has a trivial kernel. In that case $x = (A^T A)^{-1} A^T b$. The matrix $A(A^T A)^{-1} A^T$ is now the projection matrix onto $\text{im}(A)$. If the columns of A are orthonormal, this simplifies to $P = AA^T$.