

Homework 10: Coordinates

This homework is due on Friday, February 27, respectively on Tuesday, March 3, 2015 (do this early as this is exam day).

- 1 What are the \mathcal{B} -coordinates of the vector \vec{v} in the basis \mathcal{B} .

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\} ?$$

- 2 What is the matrix B for the transformation $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ in the

basis $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.

- 3 a) Find the matrix A which belongs to a reflection at the plane $x + y + 2z = 0$.

- b) Find the matrix A which belongs to the reflection at the line

spanned by $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

- 4 Find the matrix A corresponding to the orthogonal projection

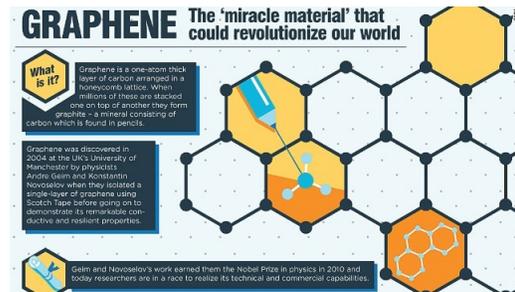
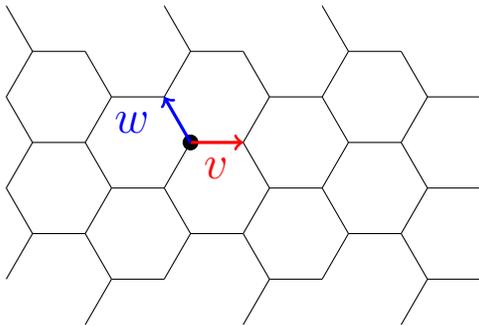
onto the plane spanned by the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

- 5 The whole plane is covered with regular hexagons "Graphene",

where the first basis vector is $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. a) Find w so that $\mathcal{B} = \{v, w\}$ is the basis as seen in the picture.

b) What are the standard coordinates of the vector given in the \mathcal{B} basis as $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$?

c) Is the point with \mathcal{B} coordinate $\begin{bmatrix} 13 \\ 21 \end{bmatrix}$ a vertex of a hexagon or the center of one?



Source: CNN

Coordinates

Given a basis $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ of a linear space V , every \vec{w} in V can be written as $\vec{w} = c_1\vec{v}_1 + \dots + c_n\vec{v}_n$, where c_i are the **coordinates** of v . The basis defines a matrix $S =$

$\left\{ \begin{bmatrix} | \\ \vec{v}_1 \\ | \end{bmatrix}, \begin{bmatrix} | \\ \vec{v}_2 \\ | \end{bmatrix} \dots \begin{bmatrix} | \\ \vec{v}_n \\ | \end{bmatrix} \right\}$. Since $S\vec{c} = \vec{w}$ we get $\boxed{\vec{c} = S^{-1}\vec{w}}$.

If A is a matrix given in the standard basis e_1, \dots, e_n and B is the matrix written in the basis \mathcal{B} , then $\boxed{B = S^{-1}AS}$.

We say B is **similar** to A . Why do we want to change basis? Because it is convenient: for example if v_1, v_2 are non-parallel vectors in a plane and v_3 is perpendicular to the plane then a projection onto the plane is the matrix

$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. The matrix in the standard basis is then $A = SBS^{-1}$.