

Homework 6: Matrix Algebra

This homework is due on Wednesday, February 18, respectively on Thursday, February 19, 2015.

1 For each pair of matrices A and B , compute both AB and BA .

a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & \frac{1}{2} \end{bmatrix}.$

b) $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 6 & 11 \end{bmatrix}.$

2 Find a 2×2 matrix A with no 0 entries such that $A^2 = 0$.

3 Find the inverse of the matrix A made from the first 4 rows of Pascal's triangle. After that, *guess without computing* the inverse of the matrix B made from the first 5 rows of Pascal's tri-

angle and verify that you have guessed right. $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix},$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}.$$

4 a) Assume $A^3 = A \cdot A \cdot A$ is the identity. Can you find a formula for A^{-1} .

b) Find a transformation in the plane which has the property that $A^7 = 1$. Find A^{-1} .

5 a) Assume A is small enough so that $B = 1 + A + A^2 + A^3 + \dots$ converges. Verify that B is the inverse of $1 - A$.

b) Let A be the 3×3 matrix which has $1/10$ in every entry. Use Mathematica to compute $1 + A + \dots + A^{10}$ and compare it with the inverse of $1 - A$.

6 Optional: lets experiment a bit! Make matrix plots of the inverses of the following 1000×1000 matrices

a) $A_{nm} = \cos(nm + 3)$

b) $A_{nm} = \cot(2.1 + n + m)$

c) $A_{nm} = 1.4 + n + m$

d) $A_{nm} = 1.1 - n^2 - m^2$.

Here is how to get a random 1000×1000 matrix in Mathematica:

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MatrixPlot [Inverse [Table [Random[] , {n, 1000} , {m, 1000} ] ] ]
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Matrix Algebra

Matrices can be added, multiplied with a scalar. One can also form the product of two matrices $A \cdot B$ as well as the inverse matrix A^{-1} if the matrix is invertible. These operations constitute the **matrix algebra**. It behaves like the algebra of real numbers but the multiplication is no more commutative in general. Besides the matrix 0 where all entries are zero there are other matrices which are not invertible. We write 1 for the identity matrix which has 1 in the diagonal and 0 everywhere else. Now $A1 = A$.

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A = {{5, 2}, {3, 4}}; MatrixInverse [A]
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MatrixPower [A, 7]
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IdentityMatrix [2] + A
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