Homework 5: transformations in geometry

This homework is due on Friday, February 12, respectively on Tuesday, February 17, 2015. (This is updated as classes on Monday were cancelled).

1 a) Find the matrix of a rotation with dilation which rotates in the plane around the origin by $45°$ counterclockwise and scales by a factor $\sqrt{2}$.

b) Find the matrix of rotation with dilation which rotates around the origin by $60°$ counterclockwise and scales by a factor 2.

2 Name the following transformations. Choose from Dilation, Shear, Rotation, Projection, Reflection, and Rotation with Dilation. (Note: not all transformations might occur.)

a) $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, b) $A = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$, c) $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$,

d) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, e) $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, f) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

3 Pair the linear transformations given by the matrices A, B, C, D, E below with one of the following types:

a) an orthogonal projection onto a line

b) reflection about a line

c) a rotation around a line

d) a reflection about a plane

e) an orthogonal projection onto a plane

$$A = \frac{1}{9} \begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix}, B = \frac{1}{9} \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{bmatrix}, C = \frac{1}{3} \begin{bmatrix} -2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & -2 \end{bmatrix},$$

$$D = \frac{1}{3} \begin{bmatrix} 2 & -2 & -1 \\ -2 & -1 & -2 \\ -1 & -2 & 2 \end{bmatrix}, E = \frac{1}{5} \begin{bmatrix} 3 & 0 & 4 \\ 0 & 5 & 0 \\ -4 & 0 & 3 \end{bmatrix}.$$

4 Assume $a^2 + b^2 = 1$. One of the three transformations is a rotation, the other is a reflection about a line, the third is an orthogonal projection onto a line. Which is which? Find the inverse in the case of rotation and reflection.

a) $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.

b) $B = \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$.

c) $C = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

5 For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\det A = ad - bc$.
Find the determinant of a shear, of a rotation, of a reflection about a line, of reflection in the origin, a projection onto the x axis, a rotation-dilation matrix with parameters (a, b).

Transformations to know

Horizontal and vertical shear, dilation (scaling), rotation (about an angle in the plane or about a line in space), orthogonal projection (onto a plane, line), reflection (about a plane, a line or point), rotation-dilation, shear dilation, projection dilation. Make your own personal collection!