

Homework 4: Linear transformations

This homework is due on Wednesday, February 11, respectively on Thursday February 12, 2015.

- 1** Which of the following transformations are linear? If it is, find the matrix A which implements the transformation.

$$\text{a) } T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2y \\ 3x \\ x \end{bmatrix} \quad \text{b) } T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 2 \\ y + 3 \\ z - 3 \end{bmatrix}$$

$$\text{c) } T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y + 2x - \sin(x) \\ x \end{bmatrix} \quad \text{d) } T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -x - y \\ z \end{bmatrix}$$

$$\text{e) } T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y + x \\ z - x \end{bmatrix}$$

- 2** Find the inverse of the following linear transformations $x \rightarrow Ax$ or state that it is not invertible

$$\text{a) } A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad \text{b) } A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\text{c) } A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \text{d) } A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$

$$\text{e) Verify that } \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad - bc) \text{ is the inverse of } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

if $ad \neq bc$.

We learn how to invert a matrix later. For now, get the inverse by solving $Ax = e_k$, rendering the k 'th column of A^{-1} .



- 3** For each of the matrices, sketch the effect of the linear transformation $T(x) = Ax$ on the face.

$$\text{a) } \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \quad \text{b) } \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}. \quad \text{c) } \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad \text{d) } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad \text{e) } \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

4 a) Let $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$. Which matrix A implements the transformation

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow v \times x = \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix} .$$

b) Which matrix A implements the transformation

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow v \cdot x = [v_1x_1 + v_2x_2 + v_3x_3] ?$$

5 Find the linear transformation which rotates counterclockwise in the plane by $3\pi/4$. Do that by drawing the images of the vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and using these vectors to build the matrix.

Main properties

If A is a matrix then the map $x \rightarrow Ax = b$ is called a **linear transformation**. It is invertible if x can be obtained uniquely from b .

The columns of the matrix play a key role. The image of the vector e_1 is the first column, the image of e_2 the second column etc.