

Homework 2: Gauss-Jordan elimination

This homework is due on Wednesday, February 4, respectively on Thursday February 5, 2015. Some problems are based on problems of Bretscher's book.

- 1 Row reduce the following matrices A, B . Try to do it in as few steps as possible. a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 3 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 16 & 20 \\ 5 & 10 & 15 & 20 & 25 \end{bmatrix}$, b) $B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

- 2 Solve the system of equations $A\vec{x} = \vec{b}$ for

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 8 \\ 27 \\ 64 \end{bmatrix}$$

by row reducing the augmented 4×5 matrix $B = [A|b]$.

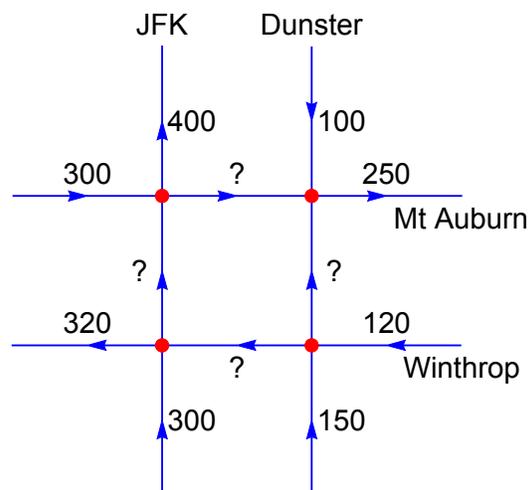
- 3 Find all solutions of the following system of equations using Gauss-Jordan elimination. As usual, you need to show all work.

$$\begin{aligned} 4x_1 + 3x_2 + 2x_3 - x_4 &= 4 \\ 5x_1 + 4x_2 + 3x_3 - x_4 &= 4 \\ -2x_1 - 2x_2 - x_3 + 2x_4 &= -3 \\ 11x_1 + 6x_2 + 4x_3 + x_4 &= 11 \end{aligned}$$

- 4 Two $n \times m$ matrices in reduced row-echelon form are called **of the same type** if they contain the same number of leading 1's in the same positions. For example, $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are

of the same type. How many types of 2×2 matrices in reduced row-echelon form are there?

- 5 The traffic in the streets near Harvard is given in the figure. Assuming that the total traffic leaving a node is the amount entering it, what can you say about the traffic at the four locations indicated by question marks? What is the highest and the lowest possible traffic volume at each location?



Main definitions

The **Gauss-Jordan Elimination** process brings a matrix into **reduced row echelon form**. It consists of **elementary row operations**: Swap two rows. Scale a row. Subtract a multiple of a row from an other. The **row-reduced matrix** $\text{rref}(A)$ has three properties:

- 1) if a row has nonzero entries, then the first nonzero entry is 1 (**this is called a leading 1**).
- 2) if a column contains a leading 1, then the other entries in that column are 0.
- 3) if a row has a leading 1, then every row above has a leading 1 to the left.

The number of leading 1 in $\text{rref}(A)$ is the **rank** of A .