

## Homework 1: Linear Equations

This homework is due on Monday, February 2, respectively Tuesday February 3, 2015. Some problems are based or modified from problems in Bretscher's book. Homework is due at the beginning of each class in the classroom.

1 Find all solutions of the linear system 
$$\begin{cases} x + 3y + 3z = 10 \\ x + 2y + 3z = 8 \\ x + y + 4z = 9 \end{cases}$$

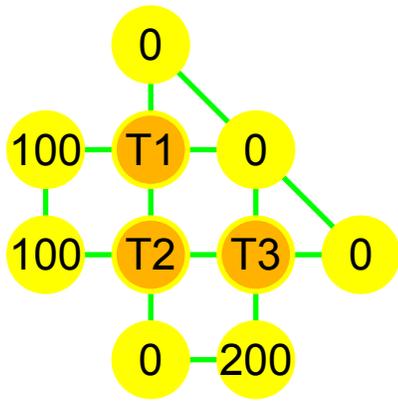
- 2 Jake and Maggie are siblings. Jake has twice as many sisters as brothers and Maggie has just as many brothers as sisters. How many kids are there in this family? (Any resemblance to the Gyllenhaal siblings is purely coincidental).



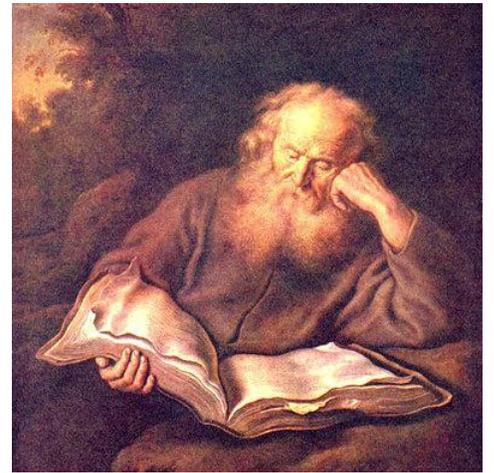
- 3 A 8km trip downstream from the Swiss waterfall "Rheinfall" to the village "Rheinau" takes 20 minutes. The return trip takes 40 minutes. How fast is the speed  $v$  (in km/h) of the boat traveling relative to the water, and how fast is the speed  $s$  (in km/h) of the river?



- 4 On a heating mesh, the temperature at exterior mesh points is 0, 100 or 200 F as given in the picture. In thermal equilibrium, each interior mesh point has the average of the temperatures at the 4 adjacent points. For example  $T_2 = (T_3 + T_1 + 100 + 0)/4$ . Find the temperatures  $T_1, T_2, T_3$ .



A ayurvedic dieter eats rice and yogurt only. Rice contains 3 grams of protein and 30 grams of carbs per serving, while yogurt contains 12 grams of protein and 20 grams carbs. If our healthy friend wants to consume 60 grams of protein and 300 grams of carbohydrates, how many servings of rice and yogurt are needed?



## Main definitions

A **linear equation** for finitely many variables  $x_1, x_2, \dots, x_n$  is an equation of the form

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b .$$

If  $m$  equations of this type are given, we have a **system of linear equations**. It can be written in matrix form  $A\vec{x} = \vec{b}$ , where  $\vec{x}$  is a column vector containing the  $n$  variables and the  $m \times n$  matrix  $A$  lists all the  $m \cdot n$  coefficients. Finally,  $\vec{b}$  is the column vector listing the values to the right.