1. The verification that $\cos(nx), \sin(nx), 1/\sqrt{2}$ form an orthonormal family is a straightforward computation when using the identities provided. For example, $\langle \cos(nx), \sin(mx) \rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin(nx)\sin(mx)\ dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos((n-m)x) - \cos((n+m)x)\ dx$ which is equal to 1 if $n = m$ and equal to 0 if $n \neq m$. The computations can be abbreviated by noting that integrating an odd 2π periodic function over $[-\pi, \pi]$ is zero.

2. To get the Fourier series of the function $f(x) = |x|$, note first that this is an **even function** so that it has a cos series. We compute

$$a_0 = \langle f, 1/\sqrt{2} \rangle = \frac{2}{\pi}\int_0^{\pi} x\frac{1}{\sqrt{2}}\ dx = \frac{\pi\sqrt{2}}{2} .$$

$$a_n = \langle f, \cos(nx) \rangle = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\ dx = \frac{2}{\pi}[\frac{\cos(n\pi)-1}{n^2}] .$$

The Fourier coefficients of $f(x) = 5 + |3x|$ are $a_0 = \frac{3\pi\sqrt{2}}{2} + 5\sqrt{2}$. and $a_n = \frac{6}{\pi}[\frac{\cos(n\pi)-1}{n^2}]$.

3. The Fourier series of $4\cos^2(3x) + 5\sin^2(11x) + 90$ is with

$$\cos^2(2x) = \frac{1+\cos(2x)}{2}$$

$$\sin^2(2x) = \frac{1-\cos(2x)}{2}$$

given as $\boxed{4/2 + 4\cos(6x)/2 - 5\cos(22x)/2 + 5/2 + 90}$. All Fourier coefficients are zero except $\boxed{a_0 = (189/2)\cdot\sqrt{2} \text{ and } a_6 = 2 \text{ and } a_{22} = -5/2}$.

4. To find the Fourier series of the function $f(x) = |\sin(x)|$, we first note that this is an **even function** so that it has a cos-series. If we integrate from 0 to π and multiply the result by 2, we can take the function $\sin(x)$ instead of $|\sin(x)|$ so that

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin(x)\cos(nx)\ dx .$$

We use one of the trigonometric identities provided in the text to solve this integral.

$$f(x) = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos(2x)}{2^2-1} + \frac{\cos(4x)}{4^2-1} + \frac{\cos(6x)}{6^2-1} + ...\right)$$

5. The square of the length of the function $f(x)$ is 1. Parceval's identity shows that

$$1 = a_0^2 + \sum_{n=1}^{\infty} a_n^2 = (\sqrt{2}\frac{2}{\pi})^2 + \frac{16}{\pi^2}[\frac{1}{(2^2-1)^2} + \frac{1}{(4^2-1)^2} + \frac{1}{(6^2-1)^2} + \cdots]$$

so that the sum is $\boxed{\pi^2/16 - 1/2}$.