

3.3.16 The third column is redundant, as we find with

$$\begin{bmatrix} 3 & 2 & -1 & 0 \\ 1 & 1 & 5 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 4 \end{bmatrix}. \text{ The fourth column, however, fails to be redundant.}$$

Thus, a basis of our kernel is  $\left( \begin{bmatrix} 3 \\ 2 \\ -1 \\ 0 \end{bmatrix} \right)$ , while  $\left( \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right)$  is a basis of our image.

3.3.26 a We notice that each of the six matrices has two identical columns. In matrices  $C$  and  $L$ , the second column is identical to the third, so that  $\ker(C) = \ker(L) = \text{span} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ . In matrices  $H, T, X$  and  $Y$ , the first column is identical to the third, so that  $\ker(H) = \ker(T) = \ker(X) = \ker(Y) = \text{span} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ . Thus, only  $L$  has the same kernel as  $C$ .

b We observe that each of the six matrices in the list has two identical rows. For example, the first and the last row of matrix  $C$  are identical, so that any vector  $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$  in  $\text{im}(C)$  will satisfy the equation  $y_1 = y_3$ . We can conclude that  $\text{im}(C) = \text{im}(H) = \text{im}(X) = \left\{ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} : y_1 = y_3 \right\}$ ,  $\text{im}(L) = \left\{ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} : y_1 = y_2 \right\}$ , and  $\text{im}(T) = \text{im}(Y) = \left\{ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} : y_2 = y_3 \right\}$ .

c Our discussion in part  $b$  shows that the answer is matrix  $L$ .

**3.3.32** We need to find all vectors  $\vec{x}$  in  $\mathbb{R}^4$  such that  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = 0$  and  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = 0$ .

This amounts to solving the system  $\begin{bmatrix} x_1 & & -x_3 & +x_4 & = & 0 \\ & x_2 & +2x_3 & +3x_4 & = & 0 \end{bmatrix}$ , which in turn amounts to finding the kernel of  $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ .

Using Kyle Numbers, we find the basis  $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix}$ .

**3.3.38 a** The rank of a  $3 \times 5$  matrix  $A$  is 0,1,2, or 3, so that  $\dim(\ker(A)) = 5 - \text{rank}(A)$  is 2,3,4, or 5.

**b** The rank of a  $7 \times 4$  matrix  $A$  is at most 4, so that  $\dim(\text{im}(A)) = \text{rank}(A)$  is 0,1,2,3, or 4.

**3.3.72**  $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

By Exercises 3.3.70 and 3.3.71a,  $[1 \ 0 \ -1 \ -2]$ ,  $[0 \ 1 \ 2 \ 3]$  is a basis of the row space of  $A$ .

**3.3.36** No; if  $\text{im}(A) = \ker(A)$  for an  $n \times n$  matrix  $A$ , then  $n = \dim(\ker(A)) + \dim(\text{im}(A)) = 2 \dim(\text{im}(A))$ , so that  $n$  is an even number.

**3.3.84** Same answer as Exercise 3.3.85.

**3.3.85** Let  $\vec{v}_1, \dots, \vec{v}_6$  be the columns of matrix  $A$ . Following the hint, we observe that  $\vec{v}_5 = 4\vec{v}_1 + 5\vec{v}_2 + 6\vec{v}_4$ , which gives the relation  $4\vec{v}_1 + 5\vec{v}_2 + 6\vec{v}_4 - \vec{v}_5 = \vec{0}$ . Thus the vector

$$\vec{x} = \begin{bmatrix} 4 \\ 5 \\ 0 \\ 6 \\ -1 \\ 0 \end{bmatrix}$$

is in the kernel of matrix  $A$ . Since  $\vec{x}$  fails to be in the kernel of matrix  $B$ , the two kernels are different, as claimed.

Ch 3.TF.2 F, by Theorem 3.3.7.

Ch 3.TF.6 F; The columns could be  $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$  in  $\mathbb{R}^5$ , for example.