Math21b Solutions Week 3 Lecture 8

3.2: 18,[30,32],40,48,54,36*,38* TF16,24*

3.2.18 Linearly dependent, since rref $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \\ 1 & 4 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. So, we find that the vector $\begin{bmatrix} 1 \\ 4 \\ 7 \\ 10 \end{bmatrix}$ turns out to be redundant.

3.2.30 $\text{im}(A) = \text{span}(\vec{e}_1, \vec{e}_2)$

We can choose $\vec{e}_1, \vec{e}_2$ as a basis of $\text{im}(A)$.

3.2.32 By inspection, the first, third and sixth columns are redundant. Thus, a basis of the image consists of the remaining column vectors: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

3.2.40 Yes; by Theorem 3.2.8, $\ker(A) = \{\vec{0}\}$ and $\ker(B) = \{\vec{0}\}$. Then $\ker(AB) = \{\vec{0}\}$ by Exercise 3.1.51, so that the columns of AB are linearly independent, by Theorem 3.2.8.

3.2.48 We can write $3x_1 + 4x_2 + 5x_3 = [3\ 4\ 5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$, so that $V = \ker[3\ 4\ 5]$.

To express V as an image, choose a basis of V, for example, $\begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -4 \end{bmatrix}$.

Then, $V = \text{im} \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & -4 \end{bmatrix}$.

There are other solutions.

3.2.54 We need to find all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = x + 2y + 3z = 0$.

These vectors have the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2s - 3t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$.

3.2.36 Yes; we know that there is a nontrivial relation $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m = \vec{0}$.

Now apply the transformation T to the vectors on both sides, and use linearity:

$T(c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m) = T(\vec{0})$, so that $c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + \cdots + c_mT(\vec{v}_m) = \vec{0}$.

This is a nontrivial relation among the vectors $T(\vec{v}_1), \ldots, T(\vec{v}_m)$, so that these vectors are linearly dependent, as claimed.

3.2.38 a Using the terminology introduced in the exercise, we need to show that any vector $\vec{v}$ in V is a linear combination of $\vec{v}_1, \ldots, \vec{v}_m$. Choose a specific vector $\vec{v}$ in V. Since we can find no more than m linearly independent vectors in V, the $m+1$ vectors $\vec{v}_1, \ldots, \vec{v}_m$, $\vec{v}$ will be linearly dependent. Since the vectors $\vec{v}_1, \ldots, \vec{v}_m$ are independent, $\vec{v}$ must be redundant, meaning that $\vec{v}$ is a linear combination of $\vec{v}_1, \ldots, \vec{v}_m$, as claimed.

b With the terminology introduced in part a, we can let $V = \text{im}\begin{bmatrix} \vec{v}_1 & \cdots & \vec{v}_m \end{bmatrix}$.

Ch 3.TF.**16** T, by Summary 3.3.10.

Ch 3.TF.**24** F; Consider $\vec{u} = \vec{e}_1$, $\vec{v} = 2\vec{e}_1$, and $\vec{w} = \vec{e}_2$.