Math21b Solutions Week 2 Lecture 4

Section 2.1: 6, [18,22], 24-30, 38, 44, 34*, 52* TF5,6*

2.1.6 Note that $x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, so that T is indeed linear, with matrix $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.

2.1.18 Compare with Exercise 16: This matrix represents a scaling by the factor of $\frac{1}{2}$; the inverse is a scaling by 2.

2.1.22 If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$, so that A represents the reflection about the $\vec{e}_1$ axis. This transformation is its own inverse: $A^{-1} = A$. (See Figure 2.4.)

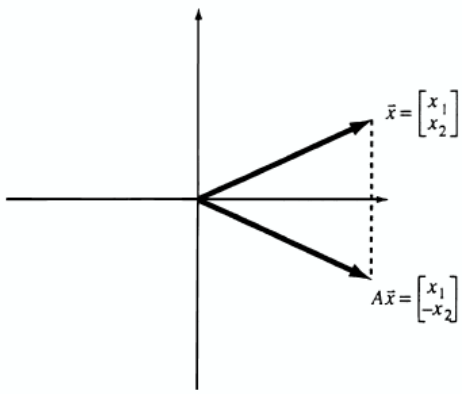

Figure 2.20: for Problem 2.1.22.

2.1.24 Compare with Example 5. (See Figure 2.5.)

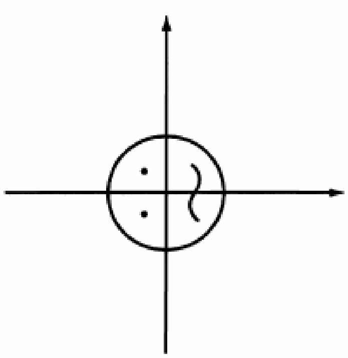

Figure 2.21: for Problem 2.1.24.

2.1.25 The matrix represents a scaling by the factor of 2. (See Figure 2.6.)

2.1.**26** This matrix represents a reflection about the line $x_2 = x_1$. (See Figure 2.7.)

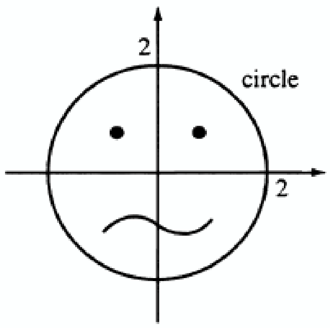

Figure 2.22: for Problem 2.1.25.

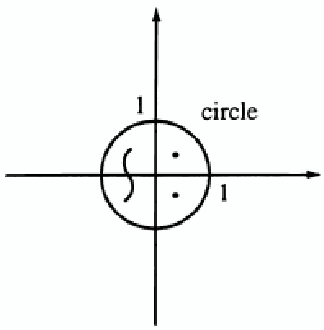

Figure 2.23: for Problem 2.1.26.

2.1.**27** This matrix represents a reflection about the $\vec{e}_1$ axis. (See Figure 2.8.)

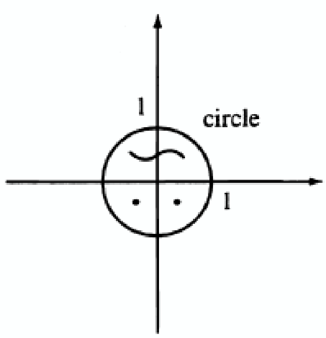

Figure 2.24: for Problem 2.1.27.

2.1.**28** If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_2 \end{bmatrix}$, so that the x_2 component is multiplied by 2, while the x_1 component remains unchanged. (See Figure 2.9.)

2.1.**29** This matrix represents a reflection about the origin. Compare with Exercise 17. (See

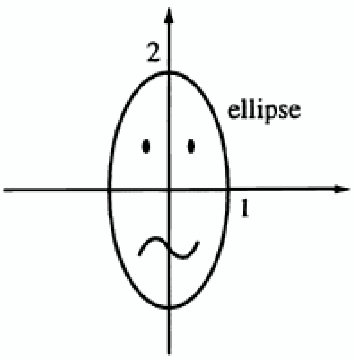

Figure 2.25: for Problem 2.1.28.

Figure 2.10.)

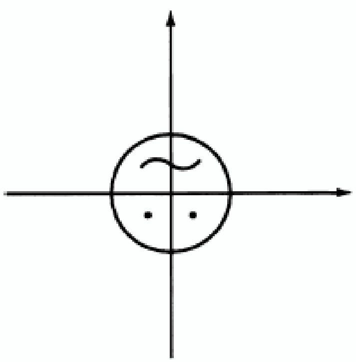

Figure 2.26: for Problem 2.1.29.

2.1.30 If $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$, so that A represents the projection onto the $\vec{e}_2$ axis. (See Figure 2.11.)

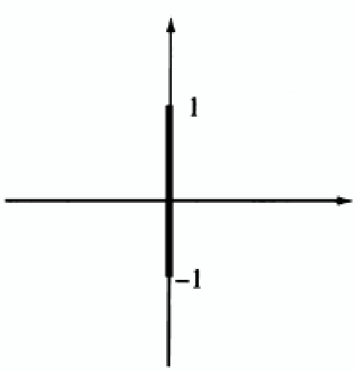

Figure 2.27: for Problem 2.1.30.

2.1.38 $T\begin{bmatrix} 2 \\ -1 \end{bmatrix} = [\vec{v}_1 \quad \vec{v}_2]\begin{bmatrix} 2 \\ -1 \end{bmatrix} = 2\vec{v}_1 - \vec{v}_2 = 2\vec{v}_1 + (-\vec{v}_2)$. (See Figure 2.15.)

2.1.44 $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, so that T is linear, with matrix $\begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$.

2.1.34 As in Exercise 2.1.33, we find $T(\vec{e}_1)$ and $T(\vec{e}_2)$; then by Theorem 2.1.2, $A = [T(\vec{e}_1)\, T(\vec{e}_2)]$. (See Figure 2.13.)

Therefore, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

2.1.52 a $A\vec{x} = \begin{bmatrix} 300 \\ 2400 \end{bmatrix}$, meaning that the total value of our money is C\$300, or, equivalently $ZAR2400$.

b From Exercise 13, we test the value $ad - bc$ and find it to be zero. Thus A is not invertible. To determine when A is consistent, we begin to compute rref $\left[A \vdots \vec{b}\right]$:

$$\begin{bmatrix} 1 & \frac{1}{8} & \vdots & b_1 \\ 8 & 1 & \vdots & b_2 \end{bmatrix} \begin{matrix} \\ -8I \end{matrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{8} & \vdots & b_1 \\ 0 & 0 & \vdots & b_2 - 8b_1 \end{bmatrix}.$$

Thus, the system is consistent only when $b_2 = 8b_1$. This makes sense, since b_2 is the total value of our money in terms of Rand, while b_1 is the value in terms of Canadian dollars. Consider the example in part a. If the system $A\vec{x} = \vec{b}$ is consistent, then there will be infinitely many solutions $\vec{x}$, representing various compositions of our portfolio in terms of Rand and Canadian dollars, all representing the same total value.

Ch 2.TF.**5** F; Matrix AB will be 3×5, by Definition 2.3.1b.

Ch 2.TF.**6** F; Note that $T\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A linear transformation transforms $\vec{0}$ into $\vec{0}$.