Math 21b Solutions: Week 1 Lecture 3
1.3: [12,14],28,36,56,62,26*,46* TF23,41*

1.3.12 $[1\ 2\ 3\ 4] \cdot \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = 1 \cdot 5 + 2 \cdot 6 + 3 \cdot 7 + 4 \cdot 8 = 70$

1.3.14 $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ or $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} =$
$\begin{bmatrix} 1 \cdot (-1) + 2 \cdot 2 + 3 \cdot 1 \\ 2 \cdot (-1) + 3 \cdot 2 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

1.3.28 There must be a leading one in each column: rref $(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

1.3.36 By Exercise 35, the ith column of A is $A\vec{e}_i$, for $i = 1, 2, 3$. Therefore, $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$.

1.3.56 We can use technology to determine that the system $\begin{bmatrix} 30 \\ -1 \\ 38 \\ 56 \\ 62 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 7 \\ 1 \\ 9 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 6 \\ 3 \\ 2 \\ 8 \end{bmatrix} +$
$x_3 \begin{bmatrix} 9 \\ 2 \\ 3 \\ 5 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ -5 \\ 4 \\ 7 \\ 9 \end{bmatrix}$ is inconsistent; therefore, the vector $\begin{bmatrix} 30 \\ -1 \\ 38 \\ 56 \\ 62 \end{bmatrix}$ fails to be a linear combination of the other four vectors.

1.3.62 We need to solve the system

$$\begin{bmatrix} 1 \\ c \\ c^2 \end{bmatrix} = x \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} + y \begin{bmatrix} 1 \\ b \\ b^2 \end{bmatrix}$$

with augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}.$$

The matrix reduces to

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c-a)(c-b) \end{bmatrix}.$$

This system is consistent if and only if $c = a$ or $c = b$. Thus the vector is a linear combination if $c = a$ or $c = b$.

1.3.26 From Example 3d we know that $\text{rank}(A) = 3$, so that $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Since all variables are leading, the system $A\vec{x} = \vec{c}$ cannot have infinitely many solutions, but it could have a unique solution (for example, if $\vec{c} = \vec{b}$) or no solutions at all (compare with Example 3c).

1.3.46 Since a, d, and f are all nonzero, we can divide the first row by a, the second row by d, and the third row by f to obtain

$$\begin{bmatrix} 1 & \frac{b}{a} & \frac{c}{a} \\ 0 & 1 & \frac{e}{d} \\ 0 & 0 & 1 \end{bmatrix}.$$

It follows that the rank of the matrix is 3.

Ch 1.TF.23 F; The system $x = 2,\ y = 3,\ x + y = 5$ has a unique solution.

Ch 1.TF.41 T; $A\vec{x} = \vec{b}$ is inconsistent if and only if rank $\left[A \vdots \vec{b}\right]$ = rank(A)+1, since there will be an extra leading one in the last column of the augmented matrix: (See Figure 1.16.)