Math21b Solutions Week 10 Lecture 25
Section 9.1: 28,32,40,42,54,24*,46*

9.1.28 $\lambda_1 = 2$, $\lambda_2 = 10$; $\vec{v}_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $c_1 = -\frac{1}{8}$, $c_2 = \frac{5}{8}$, so that $\vec{x}(t) = -\frac{1}{8}e^{2t}\begin{bmatrix} -3 \\ 2 \end{bmatrix} + \frac{5}{8}e^{10t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

9.1.32 See Exercise 26 and Figure 9.7.

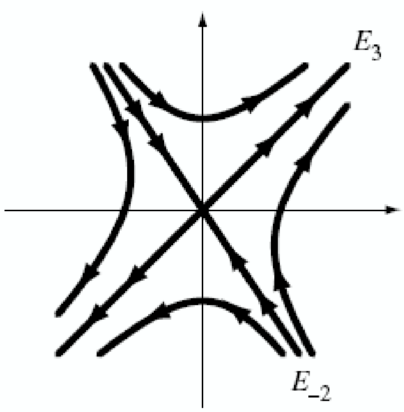


Figure 9.168: for Problem 9.1.32.

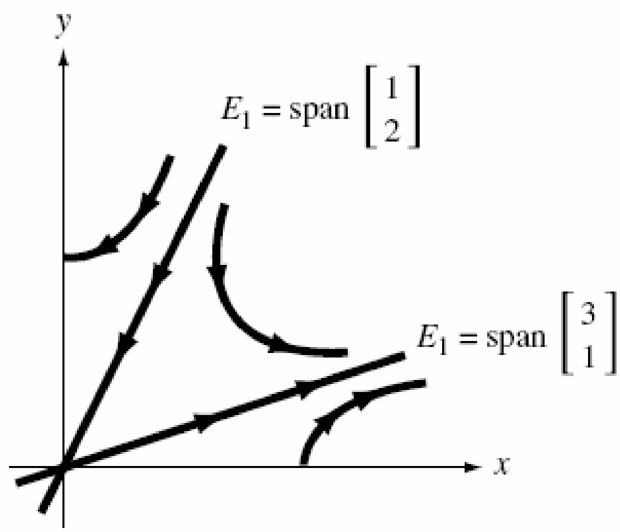
9.1.26 $\lambda_1 = 3$, $\lambda_2 = -2$; $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$, $c_1 = 5, c_2 = -1$, so that $\vec{x}(t) = 5e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - e^{-2t}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

9.1.40 $\vec{x}(t) = e^{2t}\begin{bmatrix} 2 \\ 3 \end{bmatrix} + e^{3t}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

We want a 2×2 matrix A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$ and associated eigenvectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; that is $A\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}$ or $A = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 11 & -6 \\ 12 & -6 \end{bmatrix}$.

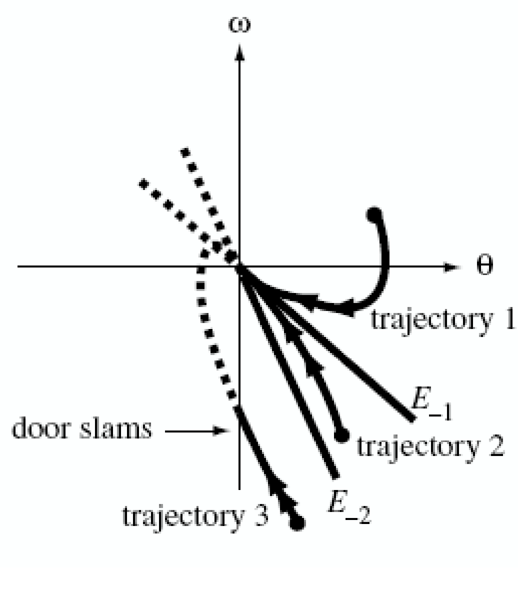
9.1.42 a The term $0.8x$ in the second equation indicates that species y is helped by x, while species x is hindered by y (consider the term $-1.2y$ in the first equation). Thus y preys on x.

b See Figure 9.16.



9.1.54 $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \lambda_1 = -1, \lambda_2 = -2; \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. See Figure 9.26.

In the case of trajectory 3 the door will slam: Initially the door is opened just a little (θ is small) and given a strong push to close it (ω is large negative). More generally, the door will slam if the point $\begin{bmatrix} \theta(0) \\ \omega(0) \end{bmatrix}$ representing the initial state is located below the line $E_{-2} = \text{span}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, that is, if $\frac{\omega(0)}{\theta(0)} < -2$.



9.1.24 We are told that $\frac{d\vec{x}}{dt} = A\vec{x}$. Let $\vec{c}(t) = e^{kt}\vec{x}(t)$. Then $\frac{d\vec{c}}{dt} = \frac{d}{dt}(e^{kt}\vec{x}) = \left(\frac{d}{dt}e^{kt}\right)\vec{x} + e^{kt}\frac{d\vec{x}}{dt} = ke^{kt}\vec{x} + e^{kt}A\vec{x} = (A + kI_n)(e^{kt}\vec{x}) = (A + kI_n)\vec{c}$, as claimed.

9.1.46 Look at the *phase portrait* in Figure 9.20.