Math 21b Solutions Week 1 Lecture 2
Section 1.2: 4,10,18,22,42,32*,20* TF6,44*

1.2.4 $\begin{bmatrix} 1 & 1 & \vdots & 1 \\ 2 & -1 & \vdots & 5 \\ 3 & 4 & \vdots & 2 \end{bmatrix} \begin{matrix} \\ -2(I) \\ -3(I) \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & -3 & \vdots & 3 \\ 0 & 1 & \vdots & -1 \end{bmatrix} \div(-3) \rightarrow \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & 1 & \vdots & -1 \\ 0 & 1 & \vdots & -1 \end{bmatrix} \begin{matrix} -II \\ \\ -II \end{matrix} \rightarrow$

$\begin{bmatrix} 1 & 0 & \vdots & 2 \\ 0 & 1 & \vdots & -1 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$, so that $\begin{matrix} x & = & 2 \\ y & = & -1 \end{matrix}$.

1.2.10 The system reduces to $\begin{bmatrix} x_1 & & & + & x_4 & = & 1 \\ & x_2 & & - & 3x_4 & = & 2 \\ & & x_3 & + & 2x_4 & = & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 & = 1 - x_4 \\ x_2 & = 2 + 3x_4 \\ x_3 & = -3 - 2x_4 \end{bmatrix}$

Let $x_4 = t$.

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - t \\ 2 + 3t \\ -3 - 2t \\ t \end{bmatrix}$, where t is an arbitrary real number.

1.2.18 a No, since the third column contains two leading ones.

b Yes

c No, since the third row contains a leading one, but the second row does not.

d Yes

1.2.22 Seven, namely $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & c \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & d \\ 0 & 1 & e \end{bmatrix}$, $\begin{bmatrix} 1 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Here, $a, b, \ldots, f$ are arbitrary constants.

1.2.42 Let x_1, x_2, x_3, and x_4 be the traffic volume at the four locations indicated in Figure 1.11.

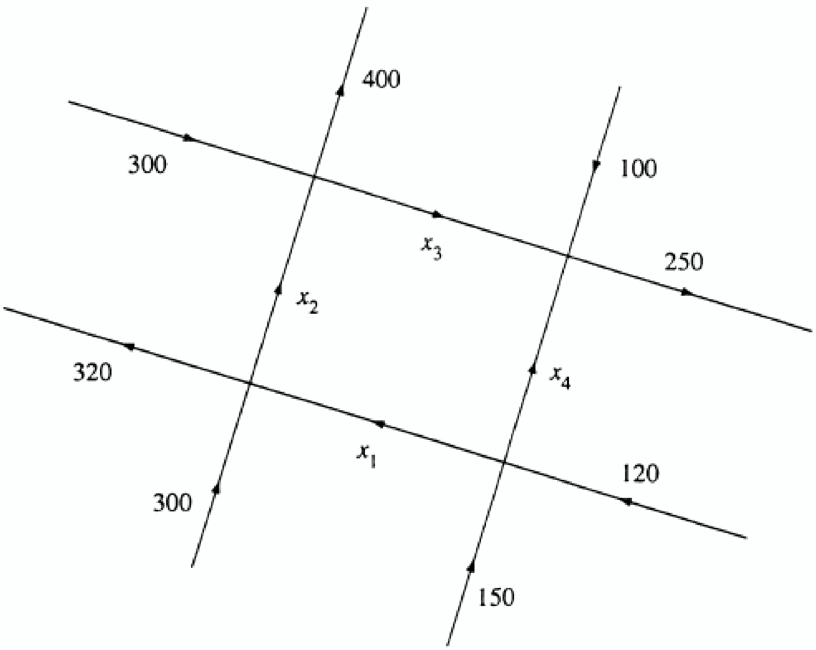

Figure 1.11: for Problem 1.2.42.

We are told that the number of cars coming into each intersection is the same as the number of cars coming out:

$$\begin{bmatrix} x_1 + 300 & = 320 + x_2 \\ x_2 + 300 & = 400 + x_3 \\ x_3 + x_4 + 100 & = 250 \\ 150 + 120 & = x_1 + x_4 \end{bmatrix} \text{ or } \begin{bmatrix} x_1 & - & x_2 & & & & & = & 20 \\ & & x_2 & - & x_3 & & & = & 100 \\ & & & & x_3 & + & x_4 & = & 150 \\ x_1 & & & & & + & x_4 & = & 270 \end{bmatrix}$$

The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 270 - t \\ 250 - t \\ 150 - t \\ t \end{bmatrix}$.

Since the x_i must be positive integers (or zero), t must be an integer with $0 \le t \le 150$.

1.2.32 The requirement $f_i'(a_i) = f_{i+1}'(a_i)$ and $f_i''(a_i) = f_{i+1}''(a_i)$ ensure that at each junction two different cubics fit "into" one another in a "smooth" way, since they must have the

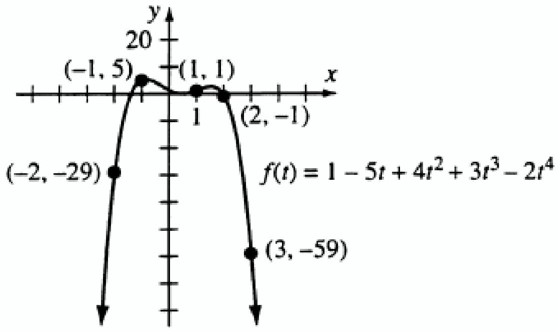

Figure 1.9: for Problem 1.2.31.

same slope and be equally curved. The requirement that $f_1'(a_0) = f_n'(a_n) = 0$ ensures that the track is horizontal at the beginning and at the end. How many unknowns are there? There are n pieces to be fit, and each one is a cubic of the form $f(t) = p + qt + rt^2 + st^3$, with p, q, r, and s to be determined; therefore, there are $4n$ unknowns. How many equations are there?

$f_i(a_i) = b_i$	for $i = 1, 2, \ldots, n$	gives n equations
$f_i(a_{i-1}) = b_{i-1}$	for $i = 1, 2, \ldots, n$	gives n equations
$f_i'(a_i) = f_{i+1}'(a_i)$	for $i = 1, 2, \ldots, n-1$	gives $n-1$ equations
$f_i''(a_i) = f_{i+1}''(a_i)$	for $i = 1, 2, \ldots, n-1$	gives $n-1$ equations
$f_1'(a_0) = 0, f_n'(a_n) = 0$		gives 2 equations

Altogether, we have $4n$ equations; convince yourself that all these equations are linear.

1.2.20 Four, namely $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (k is an arbitrary constant.)

Ch 1.TF.**6** F; As a counter-example, consider the zero matrix.

Ch 1.TF.**44** F; Consider $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. If we remove the first column, then the remaining matrix fails to be in rref.