

- 5.3.5  $3A$  will not be orthogonal, because the length of the column vectors will be 3 instead of 1, and they will fail to be unit vectors.
- 5.3.6  $-B$  will certainly be orthogonal, since the columns will be perpendicular unit vectors.
- 5.3.7  $AB$  is orthogonal by Theorem 5.3.4a.
- 5.3.8  $A+B$  will not necessarily be orthogonal, because the columns may not be unit vectors. For example, if  $A = B = I_n$ , then  $A + B = 2I_n$ , which is not orthogonal.
- 5.3.9  $B^{-1}$  is orthogonal by Theorem 5.3.4b.
- 5.3.10 This matrix will be orthogonal, by Theorem 5.3.4.
- 5.3.11  $A^T$  is orthogonal.  $A^T = A^{-1}$ , by Theorem 5.3.7, and  $A^{-1}$  is orthogonal by Theorem 5.3.4b.
- 5.3.13  $3A$  is symmetric, since  $(3A)^T = 3A^T = 3A$ .
- 5.3.14  $-B$  is symmetric, since  $(-B)^T = -B^T = -B$ .
- 5.3.15  $AB$  is not necessarily symmetric, since  $(AB)^T = B^T A^T = BA$ , which is not necessarily the same as  $AB$ . (Here we used Theorem 5.3.9a.)
- 5.3.16  $A + B$  is symmetric, since  $(A + B)^T = A^T + B^T = A + B$ .
- 5.3.17  $B^{-1}$  is symmetric, because  $(B^{-1})^T = (B^T)^{-1} = B^{-1}$ . In the first step we have used 5.3.9b.
- 5.3.18  $A^{10}$  is symmetric, since  $(A^{10})^T = (A^T)^{10} = A^{10}$ .
- 5.3.19 This matrix is symmetric. First note that  $(A^2)^T = (A^T)^2 = A^2$  for a symmetric matrix  $A$ . Now we can use the linearity of the transpose,  $(2I_n + 3A - 4A^2)^T = 2I_n^T + 3A^T - (4A^2)^T = 2I_n + 3A - 4(A^T)^2 = 2I_n + 3A - 4A^2$ .
- 5.3.20  $AB^2A$  is symmetric, since  $(AB^2A)^T = (ABBA)^T = (BA)^T(AB)^T = A^T B^T B^T A^T = AB^2A$ .

5.3.44 Note that  $A^T$  is an  $m \times n$  matrix. By Theorems 3.3.7 and 5.3.9c we have

$$\dim(\ker(A^T)) = n - \text{rank}(A^T) = n - \text{rank}(A).$$

By Theorem 3.3.6,  $\dim(\text{im}(A)) = \text{rank}(A)$ , so that  $\dim(\text{im}(A)) + \dim(\ker(A^T)) = n$ .

5.3.48 As suggested, we consider the  $QR$  factorization

$$A^T = PR$$

of  $A^T$ , where  $P$  is orthogonal and  $R$  is upper triangular with positive diagonal entries.

By Theorem 5.3.9a,  $A = (PR)^T = R^T P^T$ .

Note that  $L = R^T$  is lower triangular and  $Q = P^T$  is orthogonal.

5.3.46 By Theorem 5.2.2, the columns  $\vec{u}_1, \dots, \vec{u}_m$  of  $Q$  are orthonormal. Therefore,  $Q^T Q = I_m$ , since the  $ij$ th entry of  $Q^T Q$  is  $\vec{u}_i \cdot \vec{u}_j$ .

If we multiply the equation  $M = QR$  by  $Q^T$  from the left then  $Q^T M = Q^T QR = R$ , as claimed.

Ch 5.TF.22 T, by Theorem 5.4.1.

Ch 5.TF.23 T, by Theorem 5.4.2a.