Math21b Solutions Week 6 Lecture 14
Section 5.2: 10,14,18,36,40,44*,38* TF29*/48*

5.2.10 $\vec{u}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$$\vec{u}_2 = \frac{\vec{v}_2^{\perp}}{\|\vec{v}_2^{\perp}\|} = \frac{\vec{v}_2-(\vec{u}_1\cdot\vec{v}_2)\vec{u}_1}{\|\vec{v}_2-(\vec{u}_1\cdot\vec{v}_2)\vec{u}_1\|} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

5.2.14 $\vec{u}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{10}\begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}$

$$\vec{u}_2 = \frac{\vec{v}_2^{\perp}}{\|\vec{v}_2^{\perp}\|} = \frac{\vec{v}_2-(\vec{u}_1\cdot\vec{v}_2)\vec{u}_1}{\|\vec{v}_2-(\vec{u}_1\cdot\vec{v}_2)\vec{u}_1\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_3 = \frac{\vec{v}_3^{\perp}}{\|\vec{v}_3^{\perp}\|} = \frac{\vec{v}_3-(\vec{u}_1\cdot\vec{v}_3)\vec{u}_1-(\vec{u}_2\cdot\vec{v}_3)\vec{u}_2}{\|\vec{v}_3-(\vec{u}_1\cdot\vec{v}_3)\vec{u}_1-(\vec{u}_2\cdot\vec{v}_3)\vec{u}_2\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

5.2.18 $Q = \frac{1}{5}\begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & -5 \\ 5 & -4 & 0 \end{bmatrix}, R = \begin{bmatrix} 5 & 5 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

5.2.36 Write $M = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & -4 & 6 \\ 0 & 0 & 7 \end{bmatrix}$

$$\underset{Q_0}{\uparrow} \qquad \underset{R_0}{\uparrow}$$

This is almost the QR factorization of M: the matrix Q_0 has orthonormal columns and R_0 is upper triangular; the only problem is the entry -4 on the diagonal of R_0. Keeping in mind how matrices are multiplied, we can change all the signs in the second column of Q_0 and in the second row of R_0 to fix this problem:

$$M = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & -6 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\underset{Q}{\uparrow} \qquad \underset{R}{\uparrow}$$

5.2.40 If $\vec{v}_1, \ldots, \vec{v}_n$ are the columns of A, then $Q = \left[\frac{\vec{v}_1}{\|\vec{v}_1\|} \quad \cdots \quad \frac{\vec{v}_n}{\|\vec{v}_n\|} \right]$ and $R = \begin{bmatrix} \|\vec{v}_1\| & & 0 \\ & \ddots & \\ 0 & & \|\vec{v}_n\| \end{bmatrix}$.

(See Exercise 38 as an example.)

5.2.44 No! If m exceeds n, then there is no $n \times m$ matrix Q with orthonormal columns (if the columns of a matrix are orthonormal, then they are linearly independent).

5.2.38 Since $\vec{v}_1 = 2\vec{e}_3$, $\vec{v}_2 = -3\vec{e}_1$ and $\vec{v}_3 = 4\vec{e}_4$ are orthogonal, we have

$$Q = \left[\frac{\vec{v}_1}{\|\vec{v}_1\|} \quad \frac{\vec{v}_2}{\|\vec{v}_2\|} \quad \frac{\vec{v}_3}{\|\vec{v}_3\|} \right] = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } R = \begin{bmatrix} \|\vec{v}_1\| & 0 & 0 \\ 0 & \|\vec{v}_2\| & 0 \\ 0 & 0 & \|\vec{v}_3\| \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

Ch 5.TF.29 F. The columns fail to be unit vectors (use Theorem 5.3.3b)

Ch 5.TF.48 F; A direct computation or a geometrical argument shows that $Q = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, representing a reflection, not a rotation.