Math21b Solutions Week 4 Lecture 10
3.4: 10,16,42,50,64,32*,26* TF24,34*

3.4.10 Proceeding as in Example 1, we find $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

3.4.16 We reduce $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 1 \\ 1 & 3 & 6 & 3 \end{array}\right]$ to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & -22 \\ 0 & 0 & 1 & 8 \end{array}\right]$,

revealing that $\vec{x} = 21\vec{v}_1 - 22\vec{v}_2 + 8\vec{v}_3$, and $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 21 \\ -22 \\ 8 \end{bmatrix}$.

3.4.42 From Exercise 38, we deduce that one of our vectors should be perpendicular to this plane, while two should fall inside it. Finding the perpendicular is not difficult: we simply take the coefficient vector: $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$. Then we add two linearly independent vectors on the plane, $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, for instance. These three vectors form one possible basis.

3.4.50 a $\overrightarrow{OP} = \vec{w} + 2\vec{v}$, so that $[\overrightarrow{OP}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\overrightarrow{OQ} = \vec{v} + 2\vec{w}$, so that $[\overrightarrow{OQ}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

b $\overrightarrow{OR} = 3\vec{v} + 2\vec{w}$. See Figure 3.7.

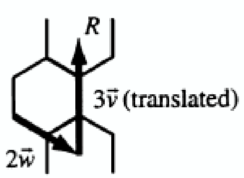

Figure 3.70: for Problem 3.4.50.

c If the tip of $\vec{u}$ is a vertex, then so is the tip of $\vec{u} + 3\vec{v}$ and also the tip of $\vec{u} + 3\vec{w}$ (draw a sketch!). We know that the tip P of $2\vec{v} + \vec{w}$ is a vertex (see part a.). Therefore, the tip S of $\overrightarrow{OS} = 17\vec{v} + 13\vec{w} = (2\vec{v} + \vec{w}) + 5(3\vec{v}) + 4(3\vec{w})$ is a vertex as well.

3.4.64 If b and c are both zero, then the given matrices are equal, so that they are similar, by Theorem 3.4.6.a. Let's now assume that at least one of the scalars b and c is nonzero; reversing the roles of b and c if necessary, we can assume that $c \neq 0$.

Let's find the matrices $S = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$ such that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & y \\ z & t \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix}$, or

$\begin{bmatrix} ax+bz & ay+bt \\ cx+bz & cy+dt \end{bmatrix} = \begin{bmatrix} ax+by & cx+dy \\ az+bt & cz+dt \end{bmatrix}$. The solutions are of the form

$S = \begin{bmatrix} \frac{(a-d)z+b}{c} & z \\ z & t \end{bmatrix}$, where z and t are arbitrary constants. Since there are *invertible* solutions S (for example, let $z = 1, t = 0$), the matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ are indeed similar.

3.4.32 Here we will build B column-by-column:

$B = [\,[T(\vec{v}_1)]_{\mathcal{B}} \quad [T(\vec{v}_2)]_{\mathcal{B}} \quad [T(\vec{v}_3)]_{\mathcal{B}}\,]$

$= [\,[\vec{v}_1 \times \vec{v}_3]_{\mathcal{B}} \quad [\vec{v}_2 \times \vec{v}_3]_{\mathcal{B}} \quad [\vec{v}_3 \times \vec{v}_3]_{\mathcal{B}}\,] = [\,[-\vec{v}_2]_{\mathcal{B}} \quad [\vec{v}_1]_{\mathcal{B}} \quad \vec{0}\,]$, since all three are perpendicular unit vectors.

So, $B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

3.4.26 Let's build B "column-by-column":

$B = [[T(\vec{v}_1)]_{\mathcal{B}}[T(\vec{v}_2)]_{\mathcal{B}}]$

$= \left[\left[\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right]_{\mathcal{B}} \quad \left[\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right]_{\mathcal{B}} \right]$

$= \left[\begin{bmatrix} 2 \\ 8 \end{bmatrix}_{\mathcal{B}} \quad \begin{bmatrix} 1 \\ 5 \end{bmatrix}_{\mathcal{B}} \right] = \begin{bmatrix} 6 & 4 \\ -4 & -3 \end{bmatrix}$.

Ch 3.TF.**24** F; Consider $\vec{u} = \vec{e}_1$, $\vec{v} = 2\vec{e}_1$, and $\vec{w} = \vec{e}_2$.

Ch 3.TF.**34** F; The identity matrix is similar only to itself.