Math21b Solutions Week 1 Lecture 1
Section 1.1: 8,24,44,46,48,20*,36* TF4,23*

1.1.8 $\begin{bmatrix} x+2y+3z & =0 \\ 4x+5y+6z & =0 \\ 7x+8y+10z & =0 \end{bmatrix} \begin{matrix} \\ -4(I) \\ -7(I) \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =0 \\ -3y-6z & =0 \\ -6y-11z & =0 \end{bmatrix} \div(-3) \rightarrow$

$$\begin{bmatrix} x+2y+3z & =0 \\ y+2z & =0 \\ -6y-11z & =0 \end{bmatrix} \begin{matrix} -2(II) \\ \\ +6(II) \end{matrix} \rightarrow \begin{bmatrix} x-z & =0 \\ y+2z & =0 \\ z & =0 \end{bmatrix} \begin{matrix} +III \\ -2(III) \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x & =0 \\ y & =0 \\ z & =0 \end{bmatrix},$$

so that $(x,y,z)=(0,0,0)$.

1.1.24 Let v be the speed of the boat relative to the water, and s be the speed of the stream; then the speed of the boat relative to the land is $v+s$ downstream and $v-s$ upstream. Using the fact that (distance) = (speed)(time), we obtain the system

$$\begin{bmatrix} 8 & =(v+s)\frac{1}{3} \\ 8 & =(v-s)\frac{2}{3} \end{bmatrix} \quad \begin{matrix} \leftarrow \text{downstream} \\ \leftarrow \text{upstream} \end{matrix}$$

The solution is $v=18$ and $s=6$.

1.1.44 Let b = Boris' money, m = Marina's money, and c = cost of a chocolate bar.

We are told that $\begin{bmatrix} b+\frac{1}{2}m & =c \\ \frac{1}{2}b+m & =2c \end{bmatrix}$, with solution $(b,m)=(0,2c)$.

Boris has no money.

1.1.46 a We set up two equations here, with our variables: x_1 = servings of rice, x_2 = servings of yogurt.

So our system is: $\begin{bmatrix} 3x_1 & +12x_2 & =60 \\ 30x_1 & +20x_2 & =300 \end{bmatrix}$.

Solving this system reveals that $x_1=8, x_2=3$.

b Again, we set up our equations: $\begin{bmatrix} 3x_1 & +12x_2 & =P \\ 30x_1 & +20x_2 & =C \end{bmatrix}$,

and reduce them to find that $x_1=-\frac{P}{15}+\frac{C}{25}$, while $x_2=\frac{P}{10}-\frac{C}{100}$.

1.1.48 Let x_1, x_2, x_3 be the number of 20 cent, 50 cent, and 2 Euro coins, respectively. Then we need solutions to the system: $\begin{bmatrix} x_1 & +x_2 & +x_3 & = 1000 \\ .2x_1 & +.5x_2 & +2x_3 & = 1000 \end{bmatrix}$

this system reduces to: $\begin{bmatrix} x_1 & & -5x_3 & = -\frac{5000}{3} \\ & x_2 & +6x_3 & = \frac{8000}{3} \end{bmatrix}$.

Our solutions are then of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 - \frac{5000}{3} \\ -6x_3 + \frac{8000}{3} \\ x_3 \end{bmatrix}$. Unfortunately for the meter maids, there are no integer solutions to this problem. If x_3 is an integer, then neither x_1 nor x_2 will be an integer, and no one will ever claim the Ferrari.

1.1.20 The total demand for the product of Industry A is 1000 (the consumer demand) plus 0.1b (the demand from Industry B). The output a must meet this demand: $a = 1000 + 0.1b$.

Setting up a similar equation for Industry B we obtain the system $\begin{bmatrix} a & = 1000 + 0.1b \\ b & = 780 + 0.2a \end{bmatrix}$

or $\begin{bmatrix} a - 0.1b & = 1000 \\ -0.2a + b & = 780 \end{bmatrix}$, which yields the unique solution $a = 1100$ and $b = 1000$.

1.1.36 $f(t) = a\cos(2t) + b\sin(2t)$ and $3f(t) + 2f'(t) + f''(t) = 17\cos(2t)$.

$f'(t) = 2b\cos(2t) - 2a\sin(2t)$ and $f''(t) = -4b\sin(2t) - 4a\cos(2t)$.

So, $17\cos(2t) = 3(a\cos(2t)+b\sin(2t))+2(2b\cos(2t)-2a\sin(2t))+(-4b\sin(2t)-4a\cos(2t)) = (-4a + 4b + 3a)\cos(2t) + (-4b - 4a + 3b)\sin(2t) = (-a + 4b)\cos(2t) + (-4a - b)\sin(2t)$.

So, our system is: $\begin{bmatrix} -a + 4b = 17 \\ -4a - b = 0 \end{bmatrix}$.

This reduces to: $\begin{bmatrix} a = -1 \\ b = 4 \end{bmatrix}$.

So our function is $f(t) = -\cos(2t) + 4\sin(2t)$.

Ch 1.TF.4 F, by Theorem 1.3.1

Ch 1.TF.23 F; The system $x = 2,\ y = 3,\ x + y = 5$ has a unique solution.