Solutions

3.2.**20** $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is redundant, simply because it is the zero vector.

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is our first non-zero vector, and thus, is not redundant.

$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and is redundant.

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is not a multiple of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and is not redundant.

$\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and is redundant.

Similarly, $\begin{bmatrix} 6 \\ 7 \\ 0 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 7 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and is also redundant.

However, by inspection, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is not a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, meaning that this last vector is not redundant. Thus, the seven vectors are linearly dependent.

3.2.**28** The three column vectors are linearly independent, since $\text{rref}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix} = I_3$.

Therefore, the three columns form a basis of $\text{im}(A)(= \mathbb{R}^3)$:

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}.$

Another sensible choice for a basis of $\text{im}(A)$ is $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

3.2.**32** By inspection, the first, third and sixth columns are redundant. Thus, a basis of the image consists of the remaining column vectors: $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$.

3.2.**44** Yes; this is a special case of Exercise 40 (recall that $\ker(A) = \{\vec{0}\}$, by Theorem 3.1.7b).

3.2.**46** Solve the system $\begin{bmatrix} x_1 & + & 2x_2 & & & + & 3x_4 & + & 5x_5 & = 0 \\ & & & & x_3 & + & 4x_4 & + & 6x_5 & = 0 \end{bmatrix}$.

The solutions are of the form

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}-2s-3t-5r\\s\\-4t-6r\\t\\r\end{bmatrix} = s\begin{bmatrix}-2\\1\\0\\0\\0\end{bmatrix} + t\begin{bmatrix}-3\\0\\-4\\1\\0\end{bmatrix} + r\begin{bmatrix}-5\\0\\-6\\0\\1\end{bmatrix}.$$

The vectors $\begin{bmatrix}-2\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-3\\0\\-4\\1\\0\end{bmatrix}, \begin{bmatrix}-5\\0\\-6\\0\\1\end{bmatrix}$ span the kernel, by construction, and they are linearly independent, by Theorem 3.2.5. Therefore, the three vectors form a basis of the kernel.

3.2.**48** We can write $3x_1 + 4x_2 + 5x_3 = [3\ 4\ 5]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = 0$, so that $V = \ker[3\ 4\ 5]$.

To express V as an image, choose a basis of V, for example, $\begin{bmatrix}4\\-3\\0\end{bmatrix}, \begin{bmatrix}0\\5\\-4\end{bmatrix}$.

Then, $V = \operatorname{im}\begin{bmatrix}4 & 0\\-3 & 5\\0 & -4\end{bmatrix}$.

There are other solutions.

3.2.**54** We need to find all vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ in $\mathbb{R}^3$ such that $\begin{bmatrix}x\\y\\z\end{bmatrix} \cdot \begin{bmatrix}1\\2\\3\end{bmatrix} = x + 2y + 3z = 0$.

These vectors have the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2s - 3t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$.

Therefore, $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ is a basis of $L^{\perp}$.