Solutions

2.3.**14** $A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $BC = [14 \quad 8 \quad 2]$, $BD = [6]$, $C^2 = \begin{bmatrix} -2 & -2 & -2 \\ 4 & 1 & -2 \\ 10 & 4 & -2 \end{bmatrix}$, $CD = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, $DB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$,

$DE = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$, $EB = [5 \ 10 \ 15]$, $E^2 = [25]$

2.3.**32** Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then we want $X \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} X$, or $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, or $\begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$, meaning that $b = c = 0$. Also, we want $X \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X$, or $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, or $\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & d \\ 0 & 0 \end{bmatrix}$ so $a = d$. Thus, $X = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} = aI_2$ must be a multiple of the identity matrix. (X will then commute with any 2×2 matrix M, since $XM = aM = MX$.)

2.3.**40** $A^2 = \frac{1}{2} \begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}$, $A^3 = I_2$, $A^4 = A$. The matrix A describes a rotation by $120° = 2\pi/3$ in the counterclockwise direction. Because A^3 is the identity matrix, we know that A^{999} is the identity matrix and $A^{1001} = A^2 = A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}$.

2.3.**42** $A^n = A$. The matrix A represents a projection on the line $x = y$ spanned by the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. We have $A^{1001} = A = (1/2) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

2.3.**46** For example, $A = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, the orthogonal projection onto the line spanned by $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2.3.**48** For example, the shear $A = \begin{bmatrix} 1 & 1/10 \\ 0 & 1 \end{bmatrix}$.

2.3.66 If $X = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$ then the diagonal entries of X^3 will be a^3, c^3, and f^3. Since we want $X^3 = 0$, we must have $a = c = f = 0$. If $X = \begin{bmatrix} 0 & 0 & 0 \\ b & 0 & 0 \\ d & e & 0 \end{bmatrix}$, then a direct computation shows that $X^3 = 0$. Thus the solutions are of the form $X = \begin{bmatrix} 0 & 0 & 0 \\ b & 0 & 0 \\ d & e & 0 \end{bmatrix}$, where b, d, e are arbitrary.