Solutions

2.2.**2** By Theorem 2.2.3, this matrix is $\begin{bmatrix} \cos(60°) & -\sin(60°) \\ \sin(60°) & \cos(60°) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$.

2.2.**28** a D is a scaling, being of the form $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$.

b E is the shear, since it is the only matrix which has the proper form (Theorem 2.2.5).

c C is the rotation, since it fits Theorem 2.2.3.

d A is the projection, following the form given in Definition 2.2.1.

e F is the reflection, using Definition 2.2.2.

2.2.**30** Write $A = [\vec{v}_1 \quad \vec{v}_2]$; then $A\vec{x} = [\vec{v}_1 \quad \vec{v}_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1\vec{v}_1 + x_2\vec{v}_2$. We must choose $\vec{v}_1$ and $\vec{v}_2$ in such a way that $x_1\vec{v}_1 + x_2\vec{v}_2$ is a scalar multiple of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, for all x_1 and x_2. This is the case if (and only if) both $\vec{v}_1$ and $\vec{v}_2$ are scalar multiples of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

For example, choose $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, so that $A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$.

2.2.**34** Keep in mind that the columns of the matrix of a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^3$ are $T(\vec{e}_1)$, $T(\vec{e}_2)$, and $T(\vec{e}_3)$.

If T is the orthogonal projection onto a line L, then $T(\vec{x})$ will be on L for all $\vec{x}$ in $\mathbb{R}^3$; in particular, the three columns of the matrix of T will be on L, and therefore pairwise parallel. This is the case only for matrix B: B represents an orthogonal projection onto a line.

A reflection transforms orthogonal vectors into orthogonal vectors; therefore, the three columns of its matrix must be pairwise orthogonal. This is the case only for matrix E: E represents the reflection about a line.

2.2.**44** By Exercise 1.1.13b, $A^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}^{-1} = \frac{1}{a^2+b^2}\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$.

If A represents a rotation through θ followed by a scaling by r, then A^{-1} represents a rotation through $-\theta$ followed by a scaling by $\frac{1}{r}$. (See Figure 2.32.)

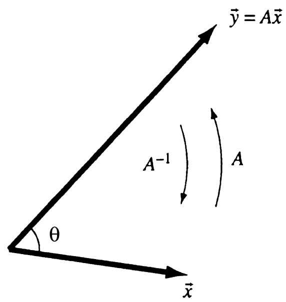

Figure 1: for Problem 2.2.44.