Solutions

2.1.**4** $A = \begin{bmatrix} 9 & 3 & -3 \\ 2 & -9 & 1 \\ 4 & -9 & -2 \\ 5 & 1 & 5 \end{bmatrix}$

2.1.**20** If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$, so that A represents the reflection about the line $x_2 = x_1$. This transformation is its own inverse: $A^{-1} = A$. (See Figure 2.2.)

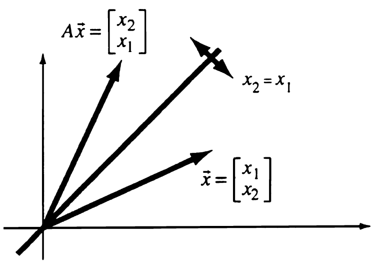

Figure 1: for Problem 2.1.20.

2.1.**22** If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$, so that A represents the reflection about the $\vec{e}_1$ axis. This transformation is its own inverse: $A^{-1} = A$. (See Figure 2.4.)

2.1.**24** Compare with Example 5. (See Figure 2.5.)

2.1.**25** The matrix represents a scaling by the factor of 2. (See Figure 2.6.)

2.1.**26** This matrix represents a reflection about the line $x_2 = x_1$. (See Figure 2.7.)

2.1.**27** This matrix represents a reflection about the $\vec{e}_1$ axis. (See Figure 2.8.)

2.1.**28** If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_2 \end{bmatrix}$, so that the x_2 component is multiplied by 2, while the x_1 component remains unchanged. (See Figure 2.9.)

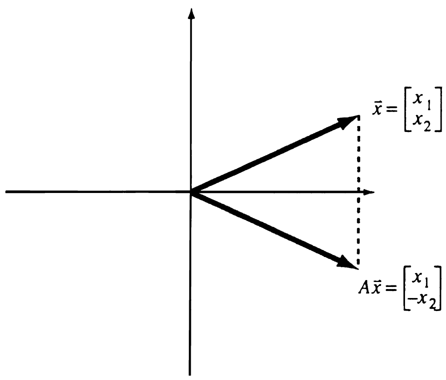

Figure 2: for Problem 2.1.22.

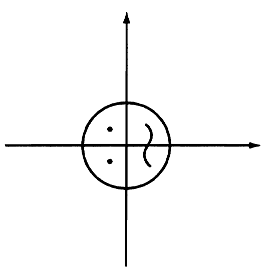

Figure 3: for Problem 2.1.24.

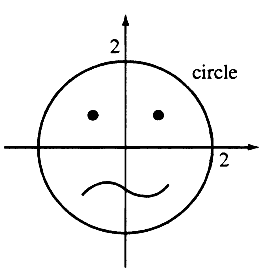

Figure 4: for Problem 2.1.25.

2.1.**29** This matrix represents a reflection about the origin. Compare with Exercise 17. (See Figure 2.10.)

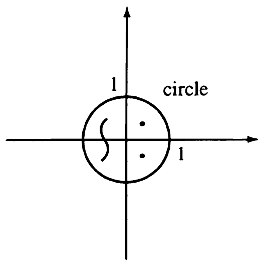

Figure 5: for Problem 2.1.26.

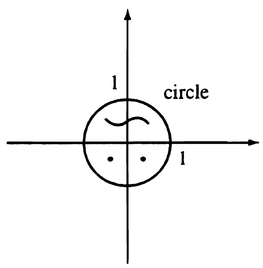

Figure 6: for Problem 2.1.27.

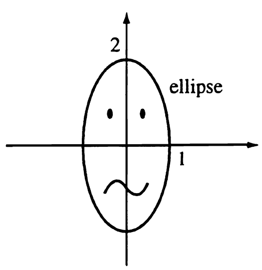

Figure 7: for Problem 2.1.28.

2.1.**30** If $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$, so that A represents the projection onto the $\vec{e}_2$ axis. (See Figure 2.11.)

2.1.**38** $T \begin{bmatrix} 2 \\ -1 \end{bmatrix} = [\vec{v}_1 \quad \vec{v}_2] \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 2\vec{v}_1 - \vec{v}_2 = 2\vec{v}_1 + (-\vec{v}_2)$. (See Figure 2.15.)

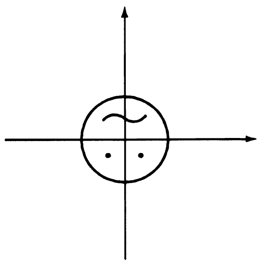

Figure 8: for Problem 2.1.29.

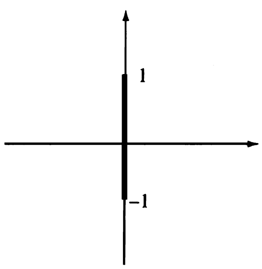

Figure 9: for Problem 2.1.30.

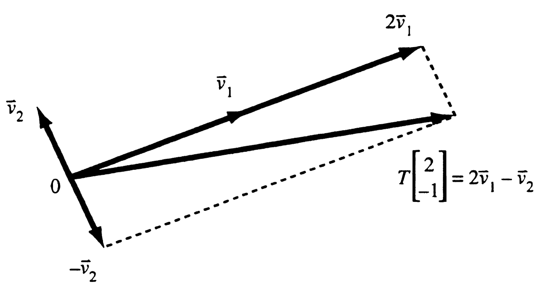

Figure 10: for Problem 2.1.38.

2.1.**52** a $A\vec{x} = \begin{bmatrix} 300 \\ 2400 \end{bmatrix}$, meaning that the total value of our money is C\$300, or, equivalently $ZAR2400$.

b From Exercise 13, we test the value $ad - bc$ and find it to be zero. Thus A is not invertible. To determine when A is consistent, we begin to compute rref $\left[A \vdots \vec{b}\right]$:

$$\left[\begin{array}{ccc} 1 & \frac{1}{8} & \vdots & b_1 \\ 8 & 1 & \vdots & b_2 \end{array}\right] \begin{array}{c} \\ -8I \end{array} \rightarrow \left[\begin{array}{ccc} 1 & \frac{1}{8} & \vdots & b_1 \\ 0 & 0 & \vdots & b_2 - 8b_1 \end{array}\right].$$

Thus, the system is consistent only when $b_2 = 8b_1$. This makes sense, since b_2 is the total value of our money in terms of Rand, while b_1 is the value in terms of Canadian dollars. Consider the example in part a. If the system $A\vec{x} = \vec{b}$ is consistent, then there will be infinitely many solutions $\vec{x}$, representing various compositions of our portfolio in terms of Rand and Canadian dollars, all representing the same total value.