Solutions

9.3.**6** Using Theorem 9.3.13, $f(t) = e^{2t}\int e^{-2t}e^{2t}\,dt = e^{2t}\int dt = e^{2t}(t+C)$, where C is an arbitrary constant.

9.3.**16** By Theorem 9.3.10, the differential equation has a particular solution of the form $f_p(t) = P\cos(t) + Q\sin(t)$. Plugging f_p into the equation we find

$(-P\cos(t) - Q\sin(t)) + 4(-P\sin(t) + Q\cos(t)) + 13(P\cos(t) + Q\sin(t)) = \cos(t)$ or

$\begin{bmatrix} 12P + 4Q = 1 \\ -4P + 12Q = 0 \end{bmatrix}$, so

$P = \frac{3}{40}$

$Q = \frac{1}{40}$.

Therefore, $f_p(t) = \frac{3}{40}\cos(t) + \frac{1}{40}\sin(t)$.

Next we find a basis of the solution space of $f''(t)+4f'(t)+13f(t) = 0$. $p_T(\lambda) = \lambda^2+4\lambda+13 = 0$ has roots $-2\pm 3i$. By Theorem 9.3.9, $f_1(t) = e^{-2t}\cos(3t)$ and $f_2(t) = e^{-2t}\sin(3t)$ is a basis of the solution space.

By Theorem 9.3.4, the solutions of the original differential equation are of the form $f(t) = c_1f_1(t) + c_2f_2(t) + f_p(t) = c_1e^{-2t}\cos(3t) + c_2e^{-2t}\sin(3t) + \frac{3}{40}\cos(t) + \frac{1}{40}\sin(t)$, where c_1, c_2 are arbitrary constants.

9.3.**29** General solution $f(t) = c_1\cos(2t)+c_2\sin(2t)+\frac{1}{3}\sin(t)$, so that $f'(t) = -2c_1\sin(2t)+2c_2\cos(2t) + \frac{1}{3}\cos(t)$ (use the approach outlined in Exercise 17)

Plug in: $0 = f(0) = c_1$ and $0 = f'(0) = 2c_2 + \frac{1}{3}$, so that $c_1 = 0$, $c_2 = -\frac{1}{6}$, and $f(t) = -\frac{1}{6}\sin(2t) + \frac{1}{3}\sin(t)$.

9.3.**43** a Using the approach of Exercise 17, we find $x(t) = c_1e^{-2t}+c_2e^{-3t}+\frac{1}{10}\cos t+\frac{1}{10}\sin t$.

b For large $t, x(t) \approx \frac{1}{10}\cos t + \frac{1}{10}\sin t$.

9.3.**44** a Using the approach of Exercises 16 and 17 we find $x(t) = e^{-2t}(c_1\cos t + c_2\sin t) - \frac{1}{40}\cos(3t) + \frac{3}{40}\sin(3t)$.

b For large $t, x(t) \approx -\frac{1}{40}\cos(3t) + \frac{3}{40}\sin(3t)$.