Solutions

9.3.**4** We can look for a sinusoidal solution $x_p(t) = P\cos(3t) + Q\sin(3t)$, as in Example 7. P and Q need to be chosen in such a way that $-3P\sin(3t) + 3Q\cos(3t) - 2P\cos(3t) - 2Q\sin(3t) = \cos(3t)$ or $\begin{bmatrix} -2P + 3Q = 1 \\ -3P - 2Q = 0 \end{bmatrix}$ with solution $P = -\frac{2}{13}$ and $Q = \frac{3}{13}$. Since the general solution of $\frac{dx}{dt} - 2x = 0$ is $x(t) = Ce^{2t}$, the general solution of $\frac{dx}{dt} - 2x = \cos(3t)$ is $x(t) = Ce^{2t} - \frac{2}{13}\cos(3t) + \frac{3}{13}\sin(3t)$, where C is an arbitrary constant.

9.3.**18** We follow the approach outlined in Exercises 16 and 17.

- Particular solution $f_p = \frac{1}{10}\cos(t) + \frac{3}{10}\sin(t)$
- Solutions of $f''(t) + 3f'(t) + 2f(t) = 0$ are $f_1(t) = e^{-t}$ and $f_2(t) = e^{-2t}$.
- The solutions of the original differential equation are of the form $f(t) = c_1e^{-t} + c_2e^{-2t} + \frac{1}{10}\cos(t) + \frac{3}{10}\sin(t)$, where c_1 and c_2 are arbitrary constants.

9.3.**28** General solution $f(t) = c_1e^{-4t} + c_2e^{3t}$, with $f'(t) = -4c_1e^{-4t} + 3c_2e^{3t}$

Plug in: $0 = f(0) = c_1 + c_2$ and $0 = f'(0) = -4c_1 + 3c_2$, so that $c_1 = c_2 = 0$ and $f(t) = 0$.

9.3.**34** a We will take downward forces as positive.

Let g = acceleration due to gravity,

ρ = density of block

a = length of edge of block

Then (weight of block) = (mass of block) $\cdot\, g$ = (density of block)(volume of block) $g = \rho a^3 g$
buoyancy = (weight of displaced water) = (mass of displaced water) $\cdot\, g$ =
(density of water) (volume of displaced water) $g = 1a^2x(t)g = a^2gx(t)$.

b Newton's Second Law of Motion tells us that

$m\frac{d^2x}{dt^2} = F =$ weight $-$ buoyancy $= \rho a^3 g - a^2gx(t)$, where $m = \rho a^3$ is the mass of the block.

$\rho a^3\frac{d^2x}{dt^2} = \rho a^3 g - a^2gx(t)$

$\frac{d^2x}{dt^2} = g - \frac{g}{\rho a}x(t)$

$\frac{d^2x}{dt^2} + \frac{g}{\rho a}x = g$

constant solution $x_p = \rho a$

general solution (use Theorem 9.3.9): $x(t) = c_1 \cos\left(\sqrt{\frac{g}{\rho a}}t\right) + c_2 \sin\left(\sqrt{\frac{g}{\rho a}}t\right) + \rho a$

Now $c_2 = 0$ since block is at rest at $t = 0$.

Plug in: $a = x(0) = c_1 + \rho a$, so that $c_1 = a - \rho a$ and

$x(t) = (a - \rho a)\cos\left(\sqrt{\frac{g}{\rho a}}t\right) + \rho a \approx 2\cos(11t) + 8$ (measured in centimeters)

c The period is $P = \frac{2\pi}{\sqrt{\frac{g}{\rho a}}} = \frac{2\pi\sqrt{\rho a}}{\sqrt{g}}$. Thus the period increases as ρ or a increases (denser wood or larger block), or as g decreases (on the moon). The period is independent of the initial state.

9.3.**40** $f_T(\lambda) = \lambda^3 + \lambda^2 - \lambda - 1 = (\lambda + 1)^2(\lambda - 1) = 0$ has roots $\lambda_{1,2} = -1,\ \lambda_3 = 1$.

In other words, we can write the differential equation as $(D + 1)^2(D - 1) = 0$.

By Exercise 38, part (d), the general solution is $x(t) = e^{-t}(c_1 + c_2t) + c_3e^t$.