Solutions

9.1.**30** $\lambda_1 = 0$, $\lambda_2 = 5$, $\vec{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $c_1 = -1$, $c_2 = 0$, so that $\vec{x}(t) = -\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

9.1.**34** See Exercise 28 and Figure 9.9.

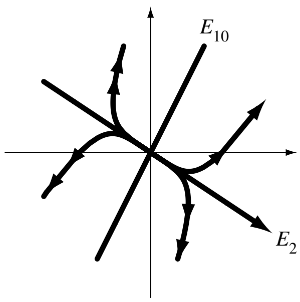

Figure 1: for Problem 9.1.34.

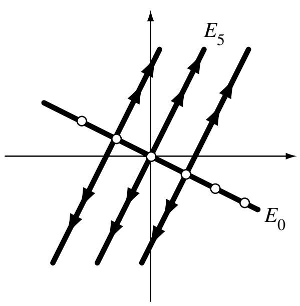

Figure 2: for Problem 9.1.35.

9.1.**40** $\vec{x}(t) = e^{2t}\begin{bmatrix} 2 \\ 3 \end{bmatrix} + e^{3t}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

We want a 2×2 matrix A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$ and associated eigenvectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; that is $A\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}$ or $A = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} =$

$$\begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 11 & -6 \\ 12 & -6 \end{bmatrix}.$$

9.1.**42** a The term $0.8x$ in the second equation indicates that species y is helped by x, while species x is hindered by y (consider the term $-1.2y$ in the first equation). Thus y preys on x.

b See Figure 9.16.

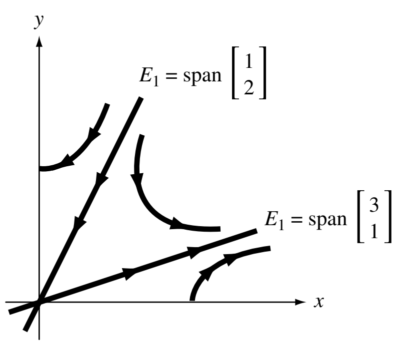

Figure 3: for Problem 9.1.42b.

c If $\frac{y(0)}{x(0)} < 2$ then both species will prosper, and $\lim_{t\to\infty} \frac{y(t)}{x(t)} = \frac{1}{3}$.

If $\frac{y(0)}{x(0)} \geq 2$ then both species will die out.

9.1.**54** $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \lambda_1 = -1, \lambda_2 = -2; \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. See Figure 9.26.

In the case of trajectory 3 the door will slam: Initially the door is opened just a little (θ is small) and given a strong push to close it (ω is large negative). More generally, the door will slam if the point $\begin{bmatrix} \theta(0) \\ \omega(0) \end{bmatrix}$ representing the initial state is located below the line $E_{-2} = \text{span}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, that is, if $\frac{\omega(0)}{\theta(0)} < -2$.

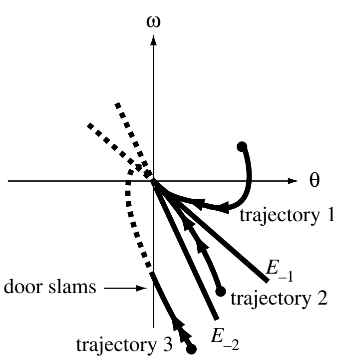

Figure 4: for Problem 9.1.54.