

Solutions

1.2.6 The system is in rref already.

$$\begin{bmatrix} x_1 & = & 3 + 7x_2 - x_5 \\ x_3 & = & 2 + 2x_5 \\ x_4 & = & 1 - x_5 \end{bmatrix}$$

Let $x_2 = t$ and $x_5 = r$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 + 7t - r \\ t \\ 2 + 2r \\ 1 - r \\ r \end{bmatrix}$$

1.2.14 The system reduces to $\begin{bmatrix} x + 2y & = & -2 \\ & z & = & 2 \end{bmatrix} \rightarrow \begin{bmatrix} x & = & -2 - 2y \\ & z & = & 2 \end{bmatrix}$.

Let $y = t$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 - 2t \\ t \\ 2 \end{bmatrix}$$

1.2.18 a No, since the third column contains two leading ones.

b Yes

c No, since the third row contains a leading one, but the second row does not.

d Yes

1.2.20 Four, namely $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (k is an arbitrary constant.)

1.2.22 Seven, namely $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & c \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & d \\ 0 & 1 & e \end{bmatrix}$, $\begin{bmatrix} 1 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$,
 $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Here, a, b, \dots, f are arbitrary constants.

1.2.32 The requirement $f'_i(a_i) = f'_{i+1}(a_i)$ and $f''_i(a_i) = f''_{i+1}(a_i)$ ensure that at each junction two different cubics fit “into” one another in a “smooth” way, since they must have the same slope and be equally curved. The requirement that $f'_1(a_0) = f'_n(a_n) = 0$ ensures that the track is horizontal at the beginning and at the end. How many unknowns are there? There are n pieces to be fit, and each one is a cubic of the form $f(t) = p + qt + rt^2 + st^3$, with p, q, r , and s to be determined; therefore, there are $4n$ unknowns. How many equations are there?

$f_i(a_i) = b_i$	for $i = 1, 2, \dots, n$	gives n equations
$f_i(a_{i-1}) = b_{i-1}$	for $i = 1, 2, \dots, n$	gives n equations
$f'_i(a_i) = f'_{i+1}(a_i)$	for $i = 1, 2, \dots, n - 1$	gives $n - 1$ equations
$f''_i(a_i) = f''_{i+1}(a_i)$	for $i = 1, 2, \dots, n - 1$	gives $n - 1$ equations
$f'_1(a_0) = 0, f'_n(a_n) = 0$		gives 2 equations

Altogether, we have $4n$ equations; convince yourself that all these equations are linear.

1.2.42 Let x_1, x_2, x_3 , and x_4 be the traffic volume at the four locations indicated in Figure 1.11.

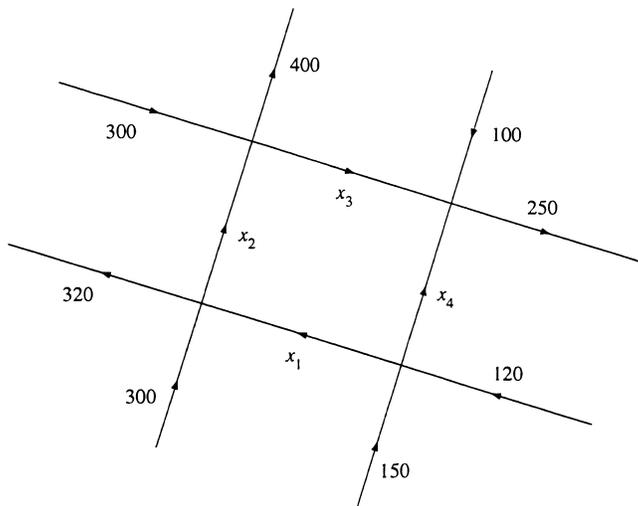


Figure 1: for Problem 1.2.42.

We are told that the number of cars coming into each intersection is the same as the number of cars coming out:

$$\begin{bmatrix} x_1 + 300 & = & 320 + x_2 \\ x_2 + 300 & = & 400 + x_3 \\ x_3 + x_4 + 100 & = & 250 \\ 150 + 120 & = & x_1 + x_4 \end{bmatrix} \text{ or } \begin{bmatrix} x_1 & - & x_2 & & = & 20 \\ & & x_2 & - & x_3 & = & 100 \\ & & & & x_3 & + & x_4 & = & 150 \\ x_1 & & & & & + & x_4 & = & 270 \end{bmatrix}$$

The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 270 - t \\ 250 - t \\ 150 - t \\ t \end{bmatrix}$.

Since the x_i must be positive integers (or zero), t must be an integer with $0 \leq t \leq 150$.

The lowest possible values are $x_1 = 120$, $x_2 = 100$, $x_3 = 0$, and $x_4 = 0$, while the highest possible values are $x_1 = 270$, $x_2 = 250$, $x_3 = 150$, and $x_4 = 150$.