Solutions

5.4.**6** Yes! For any matrix A,

$$\operatorname{im}(A) \underset{\substack{\uparrow \\ \text{Theorem 5.4.1}}}{=} (\ker(A^T))^\perp \underset{\substack{\uparrow \\ \text{Theorem 5.4.2a}}}{=} (\ker(AA^T))^\perp = (\ker(AA^T)^T)^\perp \underset{\substack{\uparrow \\ \text{Theorems 5.4.1 and 5.1.8d.}}}{=} \operatorname{im}(AA^T).$$

5.4.**16** If A is an $m \times n$ matrix, then

$$\dim(\operatorname{im} A)^\perp \underset{\substack{\uparrow \\ \text{Theorem 5.1.8c}}}{=} m - \dim(\operatorname{im} A) \underset{\substack{\uparrow \\ \text{Theorem 3.3.6}}}{=} m - \operatorname{rank}(A)$$

and $\dim(\ker(A^T)) \underset{\substack{\uparrow \\ \text{Theorem 3.3.7}}}{=} m - \operatorname{rank}(A^T)$.

It follows that $\operatorname{rank}(A) = \operatorname{rank}(A^T)$, as claimed.

5.4.**26** Here, the normal equation $A^T A\vec{x} = A^T\vec{b}$ is $\begin{bmatrix} 66 & 78 & 90 \\ 78 & 93 & 108 \\ 90 & 108 & 126 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, with solutions $\vec{x}^* = \begin{bmatrix} t - \frac{7}{6} \\ 1 - 2t \\ t \end{bmatrix}$, where t is an arbitrary constant.

5.4.**30** We attempt to solve the system

$$\begin{matrix} c_0 + 0c_1 = 0 \\ c_0 + 0c_1 = 1 \\ c_0 + 1c_1 = 1 \end{matrix}, \text{ or } \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

This system cannot be solved exactly; the least-squares solution is $\begin{bmatrix} c_0^* \\ c_1^* \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$. The line that fits the data points best is $f^*(t) = \frac{1}{2} + \frac{1}{2}t$.

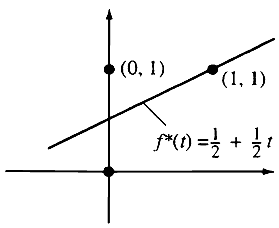

Figure 1: for Problem 5.4.30.

The line goes through the point (1, 1) and “splits the difference” between (0, 0) and (0, 1). See Figure 5.18.

5.4.**40** First we look for $\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$ such that $\log D = c_0 + c_1 \log a$.

Proceeding as in Exercise 39, we get $\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}^* \approx \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$, i.e. $\log D \approx 1.5 \log a$, hence $D \approx 10^{1.5 \log a} = a^{1.5}$.

Note that the formula $D = a^{1.5}$ is Kepler’s third law of planetary motion.