Solutions

5.3.**5** $3A$ will not be orthogonal, because the length of the column vectors will be 3 instead of 1, and they will fail to be unit vectors.

5.3.**6** $-B$ will certainly be orthogonal, since the columns will be perpendicular unit vectors.

5.3.**7** AB is orthogonal by Theorem 5.3.4a.

5.3.**8** $A+B$ will not necessarily be orthogonal, because the columns may not be unit vectors. For example, if $A = B = I_n$, then $A + B = 2I_n$, which is not orthogonal.

5.3.**9** B^{-1} is orthogonal by Theorem 5.3.4b.

5.3.**10** This matrix will be orthogonal, by Theorem 5.3.4.

5.3.**11** A^T is orthogonal. $A^T = A^{-1}$, by Theorem 5.3.7, and A^{-1} is orthogonal by Theorem 5.3.4b.

5.3.**12** This is a missing problem.

5.3.**13** $3A$ is symmetric, since $(3A)^T = 3A^T = 3A$.

5.3.**14** $-B$ is symmetric, since $(-B)^T = -B^T = -B$.

5.3.**15** AB is not necessarily symmetric, since $(AB)^T = B^T A^T = BA$, which is not necessarily the same as AB. (Here we used Theorem 5.3.9a.)

5.3.**16** $A + B$ is symmetric, since $(A + B)^T = A^T + B^T = A + B$.

5.3.**17** B^{-1} is symmetric, because $(B^{-1})^T = (B^T)^{-1} = B^{-1}$. In the first step we have used 5.3.9b.

5.3.**18** A^{10} is symmetric, since $(A^{10})^T = (A^T)^{10} = A^{10}$.

5.3.**19** This matrix is symmetric. First note that $(A^2)^T = (A^T)^2 = A^2$ for a symmetric matrix A. Now we can use the linearity of the transpose, $(2I_n + 3A - 4A^2)^T = 2I_n^T + 3A^T - (4A^2)^T = 2I_n + 3A - 4(A^T)^2 = 2I_n + 3A - 4A^2$.

5.3.**20** AB^2A is symmetric, since $(AB^2A)^T = (ABBA)^T = (BA)^T(AB)^T = A^TB^TB^TA^T = AB^2A$.

5.3.**44** Note that A^T is an $m \times n$ matrix. By Theorems 3.3.7 and 5.3.9c we have

$\dim(\ker(A^T)) = n - \text{rank}(A^T) = n - \text{rank}(A)$.

By Theorem 3.3.6, $\dim(\text{im}(A)) = \text{rank}(A)$, so that $\dim(\text{im}(A)) + \dim(\ker(A^T)) = n$.

5.3.**46** By Theorem 5.2.2, the columns $\vec{u}_1, \ldots, \vec{u}_m$ of Q are orthonormal. Therefore, $Q^TQ = I_m$, since the ijth entry of Q^TQ is $\vec{u}_i \cdot \vec{u}_j$.

If we multiply the equation $M = QR$ by Q^T from the left then $Q^TM = Q^TQR = R$, as claimed.

5.3.**48** As suggested, we consider the QR factorization

$A^T = PR$

of A^T, where P is orthogonal and R is upper triangular with positive diagonal entries.

By Theorem 5.3.9a, $A = (PR)^T = R^TP^T$.

Note that $L = R^T$ is lower triangular and $Q = P^T$ is orthogonal.