Solutions

5.1.**8** Since $\vec{u}\cdot\vec{v} = 4 - 24 + 20 = 0$, the two vectors enclose a right angle.

5.1.**10** $\vec{u}\cdot\vec{v} = 2 + 3k + 4 = 6 + 3k$. The two vectors enclose a right angle if $\vec{u}\cdot\vec{v} = 6 + 3k = 0$, that is, if $k = -2$.

5.1.**16** You may be able to find the solutions by educated guessing. Here is the systematic approach: we first find all vectors $\vec{x}$ that are orthogonal to $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$, then we identify the unit vectors among them.

Finding the vectors $\vec{x}$ with $\vec{x}\cdot\vec{v}_1 = \vec{x}\cdot\vec{v}_2 = \vec{x}\cdot\vec{v}_3 = 0$ amounts to solving the system

$$\begin{bmatrix} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - x_3 - x_4 = 0 \\ x_1 - x_2 + x_3 - x_4 = 0 \end{bmatrix}$$

$\big($we can omit all the coefficients $\frac{1}{2}\big)$.

The solutions are of the form $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t \\ -t \\ -t \\ t \end{bmatrix}$.

Since $\|\vec{x}\| = 2|t|$, we have a unit vector if $t = \frac{1}{2}$ or $t = -\frac{1}{2}$. Thus there are two possible choices for $\vec{v}_4$:

$$\begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \text{ and } \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}.$$

5.1.**20** On the line L spanned by $\vec{x}$ we want to find the vector $m\vec{x}$ closest to $\vec{y}$ (that is, we want $\|m\vec{x} - \vec{y}\|$ to be minimal). We want $m\vec{x} - \vec{y}$ to be perpendicular to L (that is, to $\vec{x}$), which means that $\vec{x}\cdot(m\vec{x} - \vec{y}) = 0$ or $m(\vec{x}\cdot\vec{x}) - \vec{x}\cdot\vec{y} = 0$ or $m = \frac{\vec{x}\cdot\vec{y}}{\vec{x}\cdot\vec{x}} \approx \frac{4182.9}{198.53^2} \approx 0.106$.

Recall that the correlation coefficient r is $r = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\|\vec{y}\|}$, so that $m = \frac{\|\vec{y}\|}{\|\vec{x}\|}r$. See Figure 5.3.

5.1.**28** Since the three given vectors in the subspace are orthogonal, we have the orthonormal basis

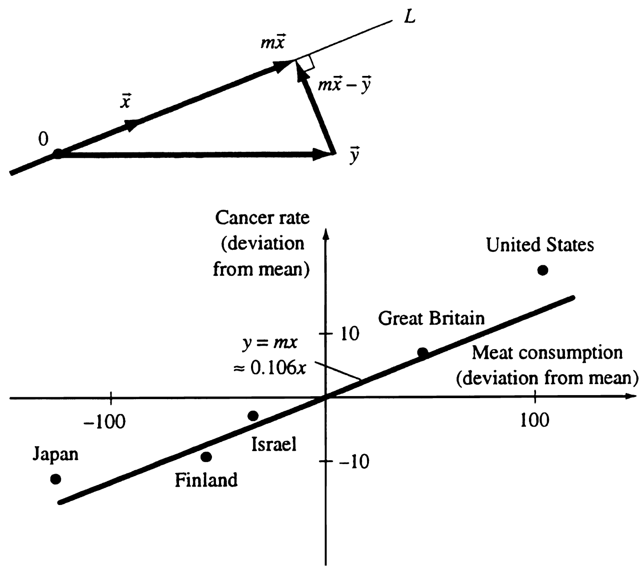

Figure 1: for Problem 5.1.20.

$$\vec{u}_1 = \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \vec{u}_2 = \frac{1}{2}\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix}, \vec{u}_3 = \frac{1}{2}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}.$$

Now we can use Theorem 5.1.5, with $\vec{x} = \vec{e}_1$: $\text{proj}_V\vec{x} = (\vec{u}_1\cdot\vec{x})\vec{u}_1 + (\vec{u}_2\cdot\vec{x})\vec{u}_2 + (\vec{u}_3\cdot\vec{x})\vec{u}_3 = \frac{1}{4}\begin{bmatrix}3\\1\\-1\\1\end{bmatrix}$.

5.1.**46** Write the projection as a linear combination of $\vec{v}_1$ and $\vec{v}_2$: $c_1\vec{v}_1 + c_2\vec{v}_2$. Now we want $\vec{v}_3 - c_1\vec{v}_1 + c_2\vec{v}_2$ to be perpendicular to V, that is, perpendicular to both $\vec{v}_1$ and $\vec{v}_2$. Using dot products, this boils down to two linear equations in two unknowns, $11 = 3c_1 + 5c_2$ and $20 = 5c_1 + 9c_2$, with the solution $c_1 = -\frac{1}{2}, c_2 = \frac{5}{2}$. Thus, the answer is $-\frac{1}{2}\vec{v}_1 + \frac{5}{2}\vec{v}_2$.

Solutions

In Exercises 1–14, we will refer to the given vectors as $\vec{v}_1, \dots, \vec{v}_m$, where $m = 1, 2$, or 3.