Solutions

4.1.**6** Not a subspace, since I_3 and $-I_3$ are invertible, but their sum is not.

4.1.**7** The set V of diagonal 3×3 matrices is a subspace of $\mathbb{R}^{3\times 3}$:

4.1.**8** This is a subspace; the justification is analogous to Exercise 7.

4.1.**9** Not a subspace; consider multiplication with a negative scalar. I_3 belongs to the set, but $-I_3$ doesn't.

4.1.**10** Let $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Let V be the set of all 3×3 matrices A such that $A\vec{v} = \vec{0}$. Then V is a subspace of $\mathbb{R}^{3\times 3}$:

4.1.**11** Not a subspace: I_3 is in rref, but the scalar multiple $2I_3$ isn't.

4.1.**36** Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. We want $AB = BA$, or

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \text{ or}$$

$$\begin{bmatrix} 2a & 3b & 3c \\ 2d & 3e & 3f \\ 2g & 3h & 3i \end{bmatrix} = \begin{bmatrix} 2a & 2b & 2c \\ 3d & 3e & 3f \\ 3g & 3h & 3i \end{bmatrix}.$$

We note that b, c, d, g must be zero, but a, e, f, h and i are chosen freely. So, our space, V, consists of all matrices of the form $\begin{bmatrix} a & 0 & 0 \\ 0 & e & f \\ 0 & h & i \end{bmatrix}$

$$= a \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + e \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + f \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + h \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + i \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus, $\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$ is a basis of V, and $\dim(V) = 5$.

4.1.**48** If $f(t) = a + bt + ct^2 + dt^3 + et^4$, then $f(-t) = a - bt + ct^2 - dt^3 + et^4$ and $-f(-t) = -a + bt - ct^2 + dt^3 - et^4$.

4.1.**48** a f is even if $f(-t) = f(t)$ for all t. Comparing coefficients we find that $b = d = 0$, so that $f(t)$ is of the form $f(t) = a + ct^2 + et^4$, with basis $1, t^2, t^4$. The dimension is 3.

b f is odd if $f(-t) = -f(t)$, which is the case if $a = c = e = 0$. The odd polynomials are of the form $f(t) = bt + dt^3$, with basis t, t^3 and dimension 2.