# Solutions

1.1.**6** $\begin{bmatrix} x+2y+3z &=& 8 \\ x+3y+3z &=& 10 \\ x+2y+4z &=& 9 \end{bmatrix} \begin{matrix} \\ -I \\ -I \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z &= 8 \\ y &= 2 \\ z &= 1 \end{bmatrix} \begin{matrix} -2(II) \\ \\ \\ \end{matrix} \rightarrow$

$\begin{bmatrix} x+3z &= 4 \\ y &= 2 \\ z &= 1 \end{bmatrix} \begin{matrix} -3(III) \\ \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x=1 \\ y=2 \\ z=1 \end{bmatrix}$, so that $(x, y, z) = (1, 2, 1)$.

1.1.**20** The total demand for the product of Industry A is the sum of the consumer demand and the demand from the industry B. This is $1000 + 0.1b$. The output is $a$. The first equation is therefore

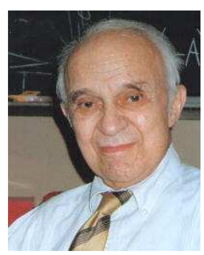
$$a = 1000 + 0.1b\ .$$

Setting up a similar equation $b = 0.2a + 780$ for Industry B gives the system

$$\begin{bmatrix} a &= 1000 + 0.1b \\ b &= 780 + 0.2a \end{bmatrix}$$

or

$$\begin{bmatrix} a - 0.1b &= 1000 \\ -0.2a + b &= 780 \end{bmatrix}\ .$$

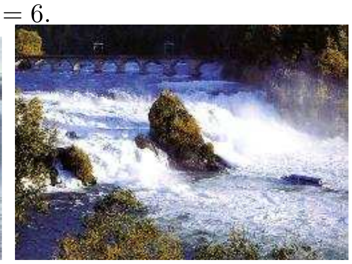


This yields to the unique solution $a = 1100$ and $b = 1000$.

1.1.**24** Let $v$ be the speed of the boat relative to the water, and $s$ be the speed of the stream; then the speed of the boat relative to the land is $v + s$ downstream and $v - s$ upstream. Using the fact that (distance) = (speed)(time), we obtain the system

$$\begin{bmatrix} 8 &= (v+s)\frac{1}{3} \\ 8 &= (v-s)\frac{2}{3} \end{bmatrix} \begin{matrix} \leftarrow \text{ downstream} \\ \leftarrow \text{ upstream} \end{matrix}$$

The solution is $v = 18$ and $s = 6$.

1.1.**36** $f(t) = a\cos(2t) + b\sin(2t)$ and $3f(t) + 2f'(t) + f''(t) = 17\cos(2t)$.

$f'(t) = 2b\cos(2t) - 2a\sin(2t)$ and $f''(t) = -4b\sin(2t) - 4a\cos(2t)$.

So, $17\cos(2t) = 3(a\cos(2t) + b\sin(2t)) + 2(2b\cos(2t) - 2a\sin(2t)) + (-4b\sin(2t) - 4a\cos(2t)) = (-4a + 4b + 3a)\cos(2t) + (-4b - 4a + 3b)\sin(2t) = (-a + 4b)\cos(2t) + (-4a - b)\sin(2t)$.

So, our system is: $\begin{bmatrix} -a + 4b = 17 \\ -4a - b = 0 \end{bmatrix}$.

This reduces to: $\begin{bmatrix} a = -1 \\ b = 4 \end{bmatrix}$.

So our function is $f(t) = -\cos(2t) + 4\sin(2t)$.

1.1.**42** We can think of the line through the points $(1, 1, 1)$ and $(3, 5, 0)$ as the intersection of any two planes through these two points; each of these planes will be defined by an equation of the form $ax + by + cz = d$. It is required that $1a + 1b + 1c = d$ and $3a + 5b + 0c = d$.

Now the system $\begin{bmatrix} a & +b & +c & -d & = & 0 \\ 3a & +5b & & -d & = & 0 \end{bmatrix}$ reduces to

$$\begin{bmatrix} a & & +\frac{5}{2}c & -2d & = & 0 \\ & b & -\frac{3}{2}c & +d & = & 0 \end{bmatrix}.$$

We can choose arbitrary real numbers for $c$ and $d$; then $a = -\frac{5}{2}c + 2d$ and $b = \frac{3}{2}c - d$. For example, if we choose $c = 2$ and $d = 0$, then $a = -5$ and $b = 3$, leading to the equation $-5x + 3y + 2z = 0$. If we choose $c = 0$ and $d = 1$, then $a = 2$ and $b = -1$, giving the equation $2x - y = 1$.

We have found one possible answer: $\begin{bmatrix} -5x & +3y & +2z & = & 0 \\ 2x & -y & & = & 1 \end{bmatrix}$.

1.1.**46** a We set up two equations here, with our variables: $x_1$ = servings of rice, $x_2$ = servings of yogurt.

So our system is: $\begin{bmatrix} 3x_1 & +12x_2 & = 60 \\ 30x_1 & +20x_2 & = 300 \end{bmatrix}$.

Solving this system reveals that $x_1 = 8, x_2 = 3$.

b Again, we set up our equations: $\begin{bmatrix} 3x_1 & +12x_2 & = P \\ 30x_1 & +20x_2 & = C \end{bmatrix}$,

and reduce them to find that $x_1 = -\frac{P}{15} + \frac{C}{25}$, while $x_2 = \frac{P}{10} - \frac{C}{100}$.