10/7/2010 FIRST HOURLY **Math 21b, Fall 2010**

Name:

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- Start by writing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly and except for problems 1-3, give details. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

1) T F The rank of A^{-1} is always equal to the rank of A if A is an invertible matrix.

2) T F $\text{rank}(A - B) = \text{rank}(A) - \text{rank}(B)$ for all 2×2 matrices.

3) T F The row reduced echelon form of an invertible 3×3 matrices is invertible.

4) T F The set of cubic polynomials $ax^3 + bx^2 + cx + d$ is a three dimensional vector space.

5) T F A system of linear equations has either 0, 1 or ∞ many solutions.

6) T F A reflection in the plane at the x axes is similar to the reflection at the y axes.

7) T F Every basis of R^3 contains exactly 3 vectors in it.

8) T F A 4×4 matrix can have $\dim(\text{im}(A)) = \dim(\ker(A))$.

9) T F The rank of a 7×3 matrix can be 4.

10) T F If $\{v_1, v_2, v_3, v_4\}$ is a set of vectors spanning a linear subspace V of $\mathbf{R}^9$, then $\dim(V) \geq 4$.

11) T F If A is a 7×5 matrix, then the dimension of $\ker(A)$ is at least 2.

12) T F The difference $A - B$ of 2 invertible 5×5 matrices A, B is invertible.

13) T F If $A\vec{x} = \vec{0}$ has a nonzero solution, where A is a 4×4 matrix, then $\text{rank}(A) \leq 3$.

14) T F If $\vec{b}$ is in $\text{im}(A)$, then $A\vec{x} = \vec{b}$ has exactly one solution.

15) T F If A and B are 2×2 matrices and $A \cdot B$ is the identity matrix 1_2, then A and B are both invertible.

16) T F If $\vec{v}$ is a nonzero vector in the kernel of A, then $\vec{v}$ is perpendicular to every row vector of A.

17) T F If $AB = I_2$ for an 2×3 matrix A and B is a 3×2 matrix, then $BA = I_3$.

18) T F If a 2×2 matrix different from the identity is its own inverse then it is a reflection at a line.

19) T F The set of vectors (x, y) in R^2 such that $|x| + y = 0$ is a linear subspace of R^2.

20) T F The space of all real 2×3 matrices is a linear space.

	Total

Problem 2) (10 points) No justifications are needed.

a) (5 points) Which of the following matrices are in row reduced echelon form?

Matrix	is in row reduced echelon form
$\begin{bmatrix} 1 & 2 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	
$\begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$	
$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$	
$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	

b) (5 points) Check the matrices which are invertible:

Matrix	invertible
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	
$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$	
$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$	
$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	
$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$	

Problem 3) (10 points) No justifications are necessary.

a) (3 points) Which of the following matrices either perform a rotation dilation or a reflection dilation? Check the corresponding boxes (it is also possible that both cases are unchecked):

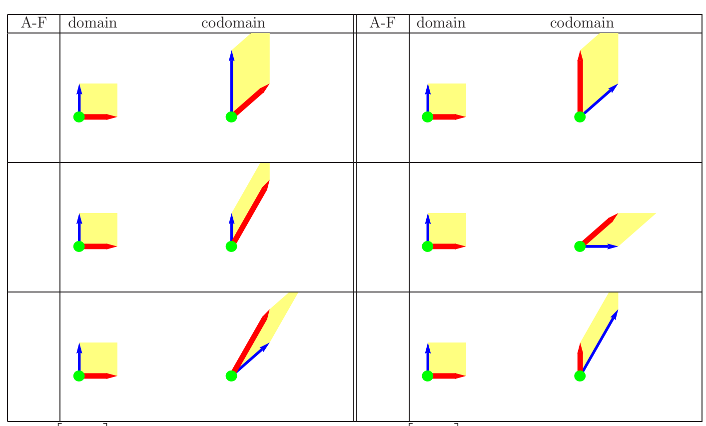
Matrix	rotation dilat.	reflection dilat.	Matrix	rotation dilat.	reflection dilat.
$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$			$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$		
$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$			$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$		

b) (2 points) Which of the following sets are linear spaces?

The space of all ...	Check
functions $f(x) = a\sin(x)$	
all 2×2 rotation matrices	

The space of all ...	Check
$(x, y, z) \in \mathbf{R}^3$ satisfying $2x + y - 4z = 1$	
smooth functions satisfying $f(x) = -f(-x^2)$	

c) (5 points) Match the matrices with their actions:



A= $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ D= $\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

B= $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ E= $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

C= $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ F= $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

Problem 4) (10 points)

a) (5 points) Find a basis of the image of the following **chess matrix**:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} .$$

b) (5 points) Find a basis for the linear subspace of all vectors in $\mathbf{R}^4$ which are perpendicular to the columns of the matrix

$$A = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix} .$$

Problem 5) (10 points)

a) (5 points) Invert the matrix

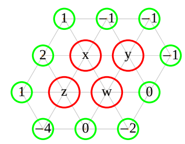
$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

by using row reduction on an augmented 4×8 matrix

b) (5 points) Find a basis for the linear space of vectors perpendicular to the kernel of

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} .$$

Problem 6) (10 points)



Ruth has a herb garden. The humidity of its soil has the property that at any given point the humidity is the sum of the neighboring humidities. Samples are taken on a hexagonal grid on 14 spots. The humidity at the four locations x, y, z, w is unknown. The corresponding equations $\left| \begin{array}{rr} x = & y+z+w+2 \\ y = & x+w-3 \\ z = & x+w-1 \\ w = & x+y+z-2 \end{array} \right|$ lead to the following problem:

Find all the solutions to the equations

$$\left| \begin{array}{rrrrrrrrr} x & - & y & - & z & - & w & = & 2 \\ -x & + & y & & & - & w & = & -3 \\ -x & & & + & z & - & w & = & -1 \\ -x & - & y & - & z & + & w & = & -2 \end{array} \right|$$

using row reduction.

Problem 7) (10 points)

a) (6 points) Find a basis of the space V of all vectors perpendicular to the three vectors

$$\{v_1, v_2, v_3\} = \{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \} .$$

b) (4 points) Use a) to find a basis for R^4 which contains v_1, v_2, v_3.

Problem 8) (10 points)

a) (5 points) The projection-dilation matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ in the basis

$$\mathcal{B} = \{v_1, v_2, v_3\} = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} , \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} , \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \}$$

is given by a matrix B. Find this 3×3 matrix B.

b) (5 points) A linear transformation T satisfies

$$T(v_1) = v_2, T(v_2) = v_3, T(v_3) = v_1$$

where v_1, v_2, v_3 are given in a). Find the matrix R implementing this transformation in the standard basis.

Problem 9) (10 points)

a) (5 points) Find A^{10} where $A = \begin{bmatrix} 4 & 3 & 0 & 0 \\ 3 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

b) (5 points) Find a 2×2 matrix X satisfying $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} X \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$