7.6.8 $\lambda_1 = 0.9, \lambda_2 = 0.8$ so $\vec{0}$ is a stable equilibrium.

7.6.14 $\lambda_1 = 0, \lambda_2 = 2k$ so $\vec{0}$ is a stable equilibrium if $|2k| < 1$ or $|k| < \frac{1}{2}$.

7.6.20 $\lambda_{1,2} = 4 \pm 3i, r = 5, \theta = \arctan\left(\frac{3}{4}\right) \approx 0.64$, so $\lambda_1 \approx 5(\cos(0.64) + i\sin(0.64)), [\vec{w}\ \vec{v}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{x}(t) \approx 5^t \begin{bmatrix} \sin(0.64t) \\ \cos(0.64t) \end{bmatrix}$. See Figure 7.37.

Spirals outwards (rotation-dilation).

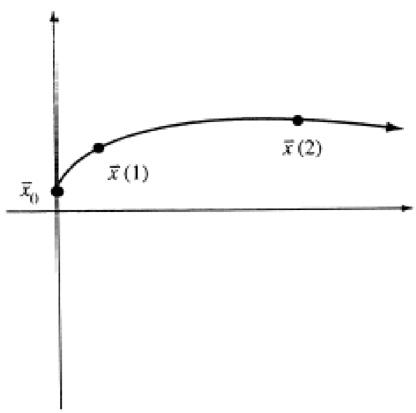

Figure 7.141: for Problem 7.6.20.

7.6.34 a If $|\det A| = |\lambda_1 \lambda_2 \cdots \lambda_n| = |\lambda_1 \lambda_2| \cdots |\lambda_n| > 1$ then at least one eigenvalue is greater than one in modulus and the zero state fails to be stable.

b If $|\det A| = |\lambda_1||\lambda_2| \cdots |\lambda_n| < 1$ we cannot conclude anything about the stability of $\vec{0}$.

$|2||0.1| < 1$ and $|0.2||0.1| < 1$ but in the first case we would not have stability, in the second case we would.

7.6.42 a $x(t+1) = x(t) - ky(t)$

$y(t+1) = kx(t) + y(t) = kx(t) + (1-k^2)y(t)$ so $\begin{bmatrix} x(t+1) \\ y(t+1) \end{bmatrix} = \begin{bmatrix} 1 & -k \\ k & 1-k^2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$.

b $f_A(\lambda) = \lambda^2 - (2-k^2)\lambda + 1 = 0$

The discriminant is $(2-k^2)^2 - 4 = -4k^2 + k^4 = k^2(k^2-4)$, which is negative if k is a small positive number ($k < 2$). Therefore, the eigenvalues are complex. By Theorem 7.6.4 the trajectory will be an ellipse, since $\det(A) = 1$.

7.6.38 a $T(\vec{v}) = A\vec{v} + \vec{b} = \vec{v}$ if $\vec{v} - A\vec{v} = \vec{b}$ or $(I_n - A)\vec{v} = \vec{b}$.

$I_n - A$ is invertible since 1 is not an eigenvalue of A. Therefore, $\vec{v} = (I_n - A)^{-1}\,\vec{b}$ is the only solution.

b Let $\vec{y}(t) = \vec{x}(t) - \vec{v}$ be the deviation of $\vec{x}(t)$ from the equilibrium $\vec{v}$.

Then $\vec{y}(t+1) = \vec{x}(t+1) - \vec{v} = A\vec{x}(t) + \vec{b} - \vec{v} = A(\vec{y}(t) + \vec{v}) + \vec{b} - \vec{v} = A\vec{y}(t) + A\vec{v} + \vec{b} - \vec{v} = A\vec{y}(t)$, so that $\vec{y}(t) = A^t\vec{y}(0)$, or $\vec{x}(t) = \vec{v} + A^t(\vec{x}_0 - \vec{v})$.

$\lim_{t\to\infty} \vec{x}(t) = \vec{v}$ for all $\vec{x}_0$ if $\lim_{t\to\infty} A^t(\vec{x}_0 - \vec{v}) = \vec{0}$. This is the case if the modulus of all the eigenvalues of A is less than 1.