7.3.14 $\lambda_1 = 0$, $\lambda_1 = \lambda_3 = 1$, eigenbasis: $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

7.3.18 $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = \lambda_4 = 1$, $E_0 = \text{span}(\vec{e}_1, \vec{e}_3)$, $E_1 = \text{span}(\vec{e}_2)$

No eigenbasis

7.3.20 For $\lambda_1 = 1$, $E_1 = \ker \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{bmatrix} = \ker \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ so if $a = 0$ then E_1 is 2-dimensional, otherwise it is 1-dimensional.

For $\lambda_2 = 2$, $E_2 = \ker \begin{bmatrix} 1 & -a & -b \\ 0 & 1 & -c \\ 0 & 0 & 0 \end{bmatrix}$ so E_2 is 1-dimensional.

Hence, there is an eigenbasis if $a = 0$.

7.3.28 Since $J_n(k)$ is triangular, its eigenvalues are its diagonal entries, hence its only eigenvalue is k. Moreover,

$$E_k = \ker(J_n(k) - kI_n) = \ker \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & \vdots \\ \vdots & \vdots & 0 & & \vdots \\ \vdots & \vdots & \vdots & & 1 \\ 0 & 0 & 0 & & 0 \end{bmatrix} = \text{span } (\vec{e}_1).$$

The geometric multiplicity of k is 1 while its algebraic multiplicity is n.

7.3.44 a $a_{11} = 0.7$ means that only 70% of the pollutant present in Lake Sils at a given time is still there a week later; some is carried down to Lake Silvaplana by the river Inn, and some is absorbed or evaporates. The other diagonal entries can be interpreted analogously: $a_{21} = 0.1$ means that 10% of the pollutant present in Lake Sils at any given time can be found in Lake Silvaplana a week later, carried down by the river Inn. The significance of the coefficient $a_{32} = 0.2$ is analogous; $a_{31} = 0$ means that no pollutant is carried down from Lake Sils to Lake St. Moritz in just one week. The matrix is lower triangular since no pollutant is carried from Lake Silvaplana to Lake Sils. The river Inn would have to flow the other way.

b The eigenvalues of A are 0.8, 0.6, 0.7, with corresponding eigenvectors

$$\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\-2\end{bmatrix}.$$

$$\vec{x}(0) = \begin{bmatrix}100\\0\\0\end{bmatrix} = 100\begin{bmatrix}0\\0\\1\end{bmatrix} - 100\begin{bmatrix}0\\1\\-1\end{bmatrix} + 100\begin{bmatrix}1\\1\\-2\end{bmatrix},$$

so $\vec{x}(t) = 100(0.8)^t\begin{bmatrix}0\\0\\1\end{bmatrix} - 100(0.6)^t\begin{bmatrix}0\\1\\-1\end{bmatrix} + 100(0.7)^t\begin{bmatrix}1\\1\\-2\end{bmatrix}$ or

$x_1(t) = 100(0.7)^t$

$x_2(t) = 100(0.7)^t - 100(0.6)^t$

$x_3(t) = 100(0.8)^t + 100(0.6)^t - 200(0.7)^t$. See Figure 7.22.

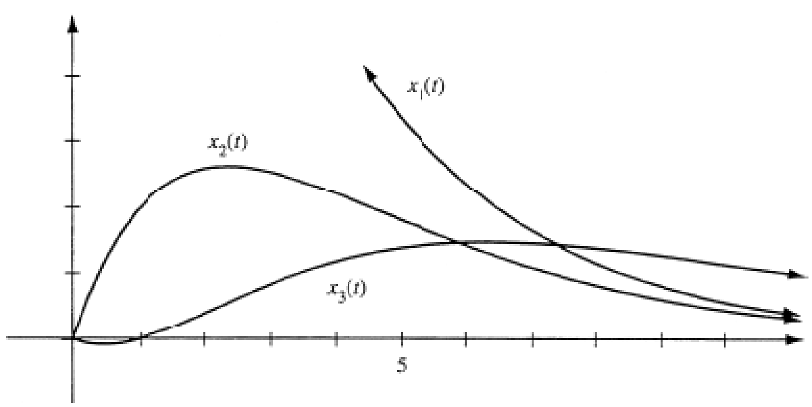

Figure 7.127: for Problem 7.3.44b.

Using calculus, we find that the function $x_2(t) = 100(0.7)^t - 100(0.6)^t$ reaches its maximum at $t \approx 2.33$. Keep in mind, however, that our model holds for integer t only.

7.3.32 Recall that a matrix and its transpose have the same rank (Theorem 5.3.9c). The geometric multiplicity of λ as an eigenvalue of A is $\dim(\ker(A-\lambda I_n)) = n-\text{rank}(A-\lambda I_n)$.

The geometric multiplicity of λ as an eigenvalue of A^T is $\dim(\ker(A^T - \lambda I_n))$

$= \dim(\ker(A - \lambda I_n)^T) = n - \text{rank}(A - \lambda I_n)^T = n - \text{rank}(A - \lambda I_n)$.

We can see that the two multiplicities are the same.

7.3.36 No, since the two matrices have different traces (see Theorem 7.3.6.d)

Ch **7.TF.25** F; Consider the zero matrix.

Ch **7.TF.51** T; If $A\vec{v} = \lambda\vec{v}$ for a nonzero $\vec{v}$, then $A^4\vec{v} = \lambda^4\vec{v} = \vec{0}$, so that $\lambda^4 = 0$ and $\lambda = 0$.